Proof:- We proof e, g and leave the others as excercises.

(e) $A \cup (B \cup C) = (A \cup B) \cup C$

$x \in A \cup (B \cup C)$ iff $x \in A$ or ($x \in B$ or $x \in C$)

iff ($x \in A$ or $x \in B$) or $x \in C$

iff $x \in (A \cup B) \cup C$.

(g) if $A \subseteq B$ then $A \cup C \subseteq B \cup C$ and $A \cap C \subseteq B \cap C$

First we proof if $A \subseteq B$ then $A \cup C \subseteq B \cup C$

Direct Suppose $x \in A \cup C \Rightarrow x \in A$ or $x \in C$

$\Rightarrow x \in B$ or $x \in C$

$\Rightarrow x \in B \cup C$

so $A \cup C \subseteq B \cup C$

Second we proof $A \subseteq B \Rightarrow A \cap C \subseteq B \cap C$

proof:- Direct: suppose $A \subseteq B$ and let $x \in A \cap C$

$\Rightarrow x \in A$ and $x \in C$

$\Rightarrow x \in B$ and $x \in C$

$\Rightarrow x \in B \cap C$.

Proposition 2.6 (Text)

Let A, B, C be subsets of some Universal set U then

a) $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$

b) $\overline{(\bar{A})} = A$.

c) $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

d) $A - B = A \cap B'$

f) $A \subseteq B$ iff $\bar{B} \subseteq \bar{A}$

<u>proof</u>:- We prove a and f and leave the others as excercise

(a) $\boxed{\subseteq}$ $\overline{A \cup B} \subseteq \bar{A} \cap \bar{B}$

So let $x \in \overline{A \cup B} \Rightarrow x \notin A \cup B$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in \bar{A}$ and $x \in \bar{B}$

$\Rightarrow x \in \bar{A} \cap \bar{B}$.

$\left\{ \begin{array}{l} x \in A \cup B \text{ iff } x \in A \text{ or } x \in B \\ x \notin A \cup B \text{ iff } x \notin A \text{ and } x \notin B \end{array} \right.$

$\bar{A} \cap \bar{B} \subseteq \overline{A \cup B}$.

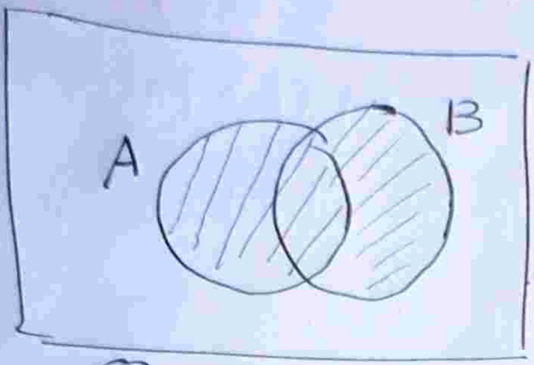
So let $x \in \bar{A} \cap \bar{B} \Rightarrow x \in \bar{A}$ and $x \in \bar{B}$

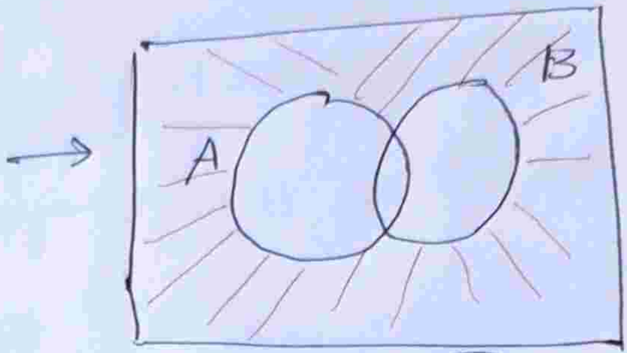
$\Rightarrow x \notin A$ and $x \notin B$.

$\Rightarrow x \notin A \cup B$.

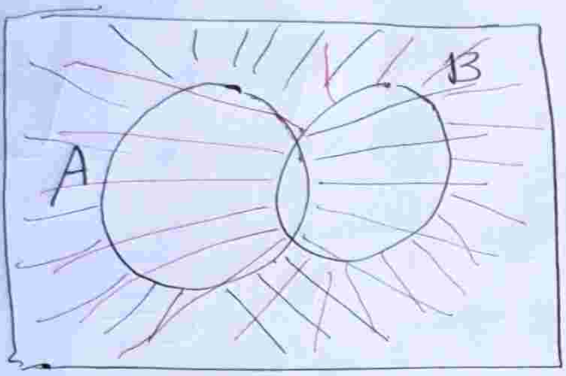
$\Rightarrow x \in \overline{A \cup B}$.

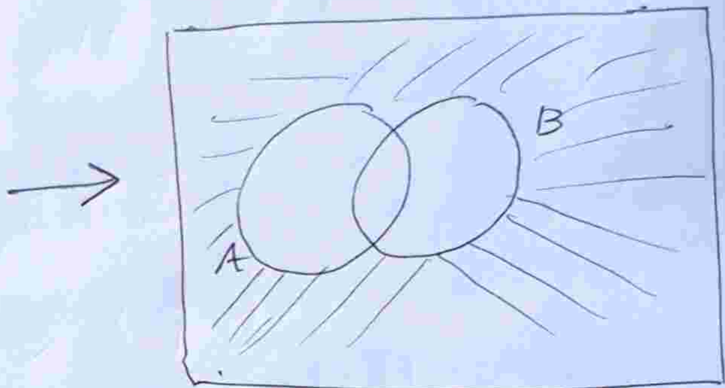
Venn Diagrams:- $\overline{A \cup B} = \overline{A} \cap \overline{B}$





① $A \cup B$ $\quad\quad$ $\overline{A \cup B}$ ②





/// $\overline{B}$ ③ $\quad\quad$ $\overline{A} \cap \overline{B}$ ④

/// $\overline{A}$

From 2 and 4 above

$$\overline{A \cup B} = \overline{A} \cap \overline{B}.$$

Theorem 2.7 | Text.

Let A, B, C be sets then

a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

We prove a and leave b as excercise.

Proof:- $\boxed{\subseteq}$ $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

$x \in A \cap (B \cup C) \Rightarrow x \in A$ and $(x \in B$ or $x \in C)$

$\Rightarrow x \in A$ and $x \in B$ Or $x \in A$ and $x \in C$

$\Rightarrow x \in A \cap B$ ~~and~~ or $x \in A \cap C$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$.

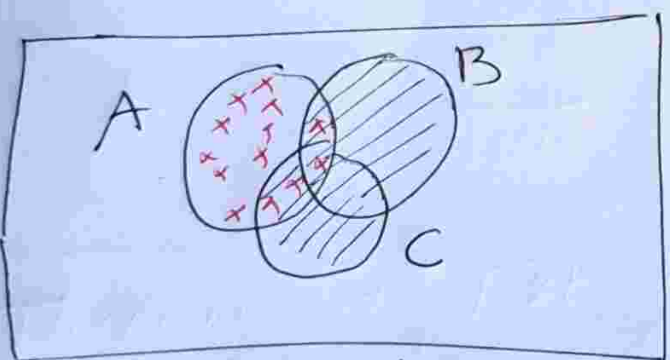
$\boxed{\supseteq}$ ~~x~~ $\in (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$

Let $x \in (A \cap B) \cup (A \cap C) \Rightarrow (x \in A$ and $x \in B)$ or $(x \in A$ and ~~x~~ $x \in C)$

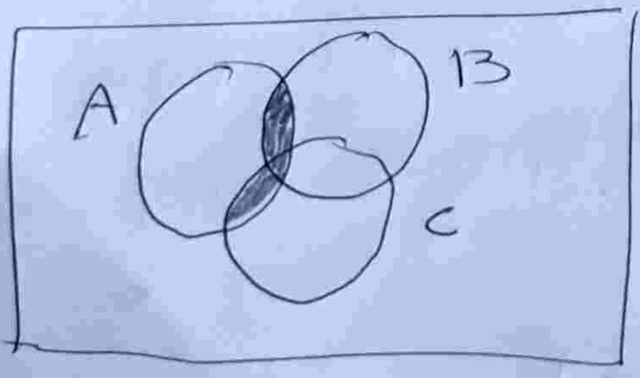
$\Rightarrow x \in A$ and $(x \in B$ or $x \in C)$

$\Rightarrow x \in A \cap (B \cup C)$.

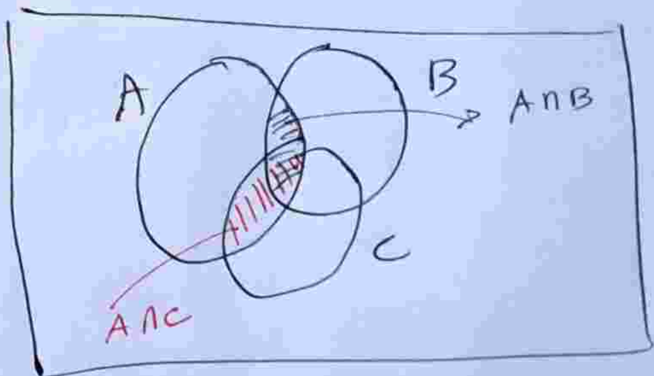
$A \cap (B \cup C)$

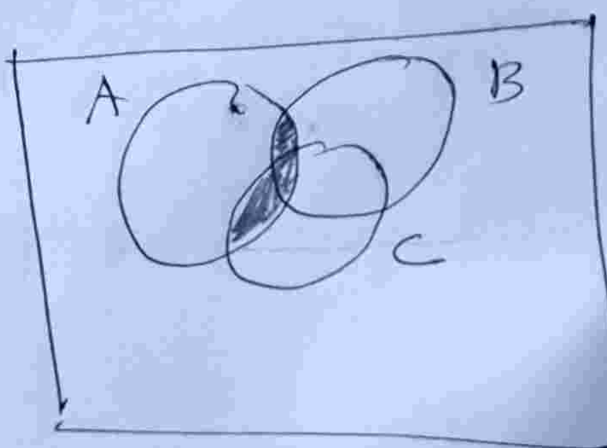


xx A   /// B∪C



$(A \cap B) \cup (A \cap C)$





2.3 Indexed Famitlies of Sets.

Ex:- Suppose $A_1 = \{1, 2, 3, 5, 6, 9\}$

$A_2 = \{2, 3, 4, 7, 8\}$

$A_3 = \{3, 4, 5, 7, 10\}$

$B = \{3, 4, 5, 6, 11, 12\}$

$I = \{1, 2, 3\}$ then

$$A_1 \cup (A_2 \cup A_3) = \bigcup_{i=1}^{3} A_i = \bigcup_{i \in I} A_i = \{1, 2, 3, 5, 6, 9, 4, 7, 8, 1$$

$$\bigcap_{i=1}^{3} A_i = \bigcap_{i \in I} A_i = \{3\}$$

$$B \cap \left(\bigcup_{i \in I} A_i\right) = \{3, 4, 5, 6\}$$

$B \cap A_1 = \{3, 5, 6\}$

$B \cap A_2 = \{3, 4\}$

$B \cap A_3 = \{3, 4, 5\}$

$$(B \cup A_1) \cup (B \cap A_2) \cup (B \cap A_3) = \bigcup_{i=1}^{3} B \cap A_i$$

$$= \{3, 4, 5, 6\}$$

so in general $B \cap \left(\bigcup_{i \in I} A_i\right) = \bigcup_{i \in I} B \cap A_i$.

So Def^n:-

$$x \in \bigcup_{i \in I} A_i \iff \exists\, i \in I \text{ s } x \in A_i$$

$$x \in \bigcap_{i \in I} A_i \iff \forall i \in I,\ x \in A_i.$$

Th 2.8 Text

Let $\mathcal{A} = \{A_\alpha : \alpha \in \Lambda\}$ be an indexed familly of sets

Let $\beta \in \Lambda$ then a) $A_\beta \subseteq \bigcup_{\alpha \in \Lambda} A_\alpha$.

b) $\bigcap_{\alpha \in \Lambda} A_\alpha \subseteq A_\beta$.

proof:- See text.

Th:- Let $\{A_\alpha : \alpha \in \Lambda\}$ be an indexed family of sets and let B be a set then

a) $B \cap \left(\bigcup_{\alpha \in \Lambda} A_\alpha \right) = \bigcup_{\alpha \in \Lambda} B \cap A_\alpha$

b) $B \cup \left(\bigcap_{\alpha \in \Lambda} A_\alpha \right) = \bigcap_{\alpha \in \Lambda} B \cap A_\alpha$.

c) $\overline{\left(\bigcap_{\alpha \in \Lambda} A_\alpha \right)} = \bigcup_{\alpha \in \Lambda} \overline{A_\alpha}$

d) $\overline{\bigcup_{\alpha \in \Lambda} A_\alpha} = \bigcap_{\alpha \in \Lambda} \overline{A_\alpha}$

proof:- Excercise (see text)