

(Q1) Consider the equation  $x = \cos x$

- (a) Use Newton's method with  $P_0 = 0.2$  to estimate the solution of this equation with error less than  $10^{-5}$
- (b) Find the order of convergence and the asymptotic error constant both numerically and theoretically.

Solution: (a)  $f(x) = x - \cos x$ ,  $f'(x) = 1 + \sin x$ ,  $f''(x) = \cos x$

Newton's iteration:  $P_{n+1} = P_n - \frac{P_n - \cos P_n}{1 + \sin P_n}$

$n$	$P_n$	$ P_n - P_{n-1} $
0	0.2	—
1	0.850777122	0.650777122
2	0.741530193	0.109246929
3	0.739086449	0.002443744
4	0.739085133	0.000001316

So  $P \approx P_4 = 0.739085133$

(b) Theoretically:  $f'(P) = f'(0.739085133) = 1.673612029 \neq 0$ ,

So  $P$  is a simple root.

Therefore,  $R = 2$  and  $A = \frac{|f''(P)|}{2|f'(P)|} = 0.220805395$

Numerically:

$ E_n  \approx  P_n - P_{n-1} $	$\frac{ E_{n+1} }{ E_n ^2}$
0.650777122	0.257955435
0.109246929	0.204756281
0.002443744	0.220365941
0.000001316	

**(Q2)** Consider the equation  $x = \cos x$ .

- (a) If  $P_0 = 0.5$  and  $P_1 = \frac{\pi}{4}$ , use the secant method to approximate the root of the equation with accuracy of  $10^{-4}$ .
- (b) Find the order of convergence and the asymptotic error constant both theoretically and numerically.

Solution: (a)  $f(x) = x - \cos x$ ,  $f'(x) = 1 + \sin x$ ,  $f''(x) = \cos x$

Secant's iteration:  $P_{n+2} = P_{n+1} - \frac{f(P_{n+1})(P_{n+1} - P_n)}{f(P_{n+1}) - f(P_n)}$

$n$	$P_n$	$ P_n - P_{n-1} $
0	0.5	—
1	0.785398163	0.285398163
2	0.736384138	0.049014025
3	0.739058138	0.002674
4	0.739085149	0.000027011338

So  $P \approx P_4 = 0.739085149$

(b) Theoretically:  $f'(P) = f'(0.739085149) = 1.673612041 \neq 0$ ,

So  $P$  is a simple root.

Therefore,  $R = 1.618$  and  $A = \left| \frac{f''(P)}{2f'(P)} \right|^{0.618} = 0.393185938$

Numerically:

$ E_n  \approx  P_n - P_{n-1} $	$\frac{ E_{n+1} }{ E_n ^{1.618}}$
0.285398163	0.372735166
0.049014025	0.351741034
0.002674	0.393005788
0.000027011338	