Math330

Quadrature Formulas

2017/2018

Mahmoud Ghannam

$$\int_{x_0}^{x_1} f(x)dx = \frac{h}{2}(f_0 + f_1) - \frac{h^3 f^{(2)}(c)}{12}$$

$$\int_{x_0}^{x_2} f(x)dx = \frac{h}{3}(f_0 + 4f_1 + f_2) - \frac{h^5 f^{(4)}(c)}{90}$$

$$\int_{x_0}^{x_3} f(x)dx = \frac{3h}{8}(f_0 + 3f_1 + 3f_2 + f_3) - \frac{3h^5 f^{(4)}(c)}{80}$$

$$\int_{x_0}^{x_4} f(x)dx = \frac{2h}{45}(7f_0 + 32f_1 + 12f_2 + 32f_3 + 7f_4) - \frac{8h^7 f^{(6)}(c)}{945}$$

$$\int_{-1}^{1} f(x)dx = 2f(0) + \frac{f''(c)}{3}$$

$$\int_{-1}^{1} f(x)dx = f(\frac{-1}{\sqrt{3}}) + f(\frac{1}{\sqrt{3}}) + \frac{f^{(4)}(c)}{135}$$

$$\int_{-1}^{1} f(x)dx = \frac{5f(-\sqrt{0.6}) + 8f(0) + 5f(\sqrt{0.6})}{9} + \frac{f^{(6)}(c)}{15750}$$

$$\int_{x_0}^{x_2} f(x)\ dx = 2hf_1 + \frac{h^3}{3}f''(c)\ ,\ \ h = \frac{x_2 - x_0}{2}$$

$$\int_{x_0}^{x_3} f(x)\ dx = \frac{3h}{2}(f_1 + f_2) + \frac{3h^3}{4}f''(c)\ ,\ \ h = \frac{x_3 - x_0}{3}$$

$$\int_{x_0}^{x_4} f(x)\ dx = \frac{4h}{3}(2f_1 - f_2 + 2f_3) + \frac{14h^5}{45}f^{(4)}(c)\ ,\ \ h = \frac{x_4 - x_0}{4}$$

$$\int_{0}^{3h} f(x)\ dx = \frac{3h}{4}(3f(h) + f(3h)) - \frac{3h^4}{8}f'''(c)$$

$$\int_{-h}^{h} f(x)\ dx = \frac{h}{2}(f(-h) + 3f(\frac{h}{3})) + \frac{2h^4}{27}f'''(c)$$

$$\int_{0}^{1} f(x)\ dx = \frac{1}{4}(f(0) + 3f(\frac{2}{3})) + \frac{f'''(c)}{216}$$

$$\int_{-1}^{2} f(x)\ dx = \frac{3}{2}(f(0) + f(1)) + \frac{3f''(c)}{4}$$

Question: Consider $\int_{-1}^{1} f(x)\ dx = Af(-1) + Bf(x_1)$. Assume DOP=2

(1) Find A, B, x_1 (Answer: $A = 1/2\ \ B = 3/2\ \ x_1 = 1/3$)

(2) Find $E[f]$ (Answer: $\frac{2}{27}f'''(c)$)