

Step 3

$$\text{Mult: } M_{43} = -\frac{1}{4}$$

$$R_4 + \frac{1}{4}R_3 \left[\begin{array}{cccc} 2 & 6 & 4 & 3 \\ 0 & -1 & -1 & 2.5 \\ 0 & 0 & 4 & -30 \\ 0 & 0 & 0 & 24 \end{array} \right]$$

Step	+	-	* , ÷
3	1(1)		1(1), 1

$$L = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 2 & 10 & 1 & 0 \\ -\frac{3}{2} & -10 & -\frac{1}{4} & 1 \end{array} \right]$$

No Cost
Mult. keeps ~~to~~

Cost of $A^{\#} = LU$

Step	+	-	* , ÷
1	3(3)		3(3) - 3
2	2(2)		2(2) - 2
3	1(1)		1(1) - 1
	14		14, 6

$$\Rightarrow \boxed{\text{Cost} = 34}$$

$$LY = b \Rightarrow \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 2 & 10 & 1 & 0 \\ -\frac{3}{2} & -10 & -\frac{1}{4} & 1 \end{array} \right) \left(\begin{array}{c} y_1 \\ y_2 \\ y_3 \\ y_4 \end{array} \right) = \left(\begin{array}{c} 22 \\ 13 \\ 20 \\ 6 \end{array} \right)$$

$$\Rightarrow y_1 = 22$$

$$\frac{1}{2}y_1 + y_2 = 13$$

$$2y_1 + 10y_2 + y_3 = 20$$

$$-\frac{3}{2}y_1 - 10y_2 - \frac{1}{4}y_3 + y_4 = 6$$

$$F.S \quad y_1 = 22$$

$$\frac{1}{=} \text{مماصة الكفاليب} \quad y_2 = 13 - \frac{1}{2}y_1$$

$$y_3 = 20 - 2y_1 - 10y_2$$

$$y_4 = 6 + \frac{3}{2}y_2 + 10y_2 + \frac{1}{4}y_3$$

$$\text{Cost of Forward Substitution} = n^2 - n \\ = 16 - 4 = 12$$

$$y_1 = 22$$

$$y_2 = 2$$

$$y_3 = -44$$

$$y_4 = 48$$

$$uX = y$$

$$2x_1 + 6x_2 + 4x_3 + 3x_4 = 22$$

$$-x_2 - x_3 + 2.5x_4 = 2$$

$$4x_3 - 30x_4 = +44$$

$$24x_4 = 48$$

$$B.S \Rightarrow \text{Cost} = n^2 = 16$$

Total Cost of Solving a 4×4 system using L.U Fac

$$\Rightarrow 34 + 12 + 16 = 64$$

Now Cost LU Factorization for $n \times n$ system.

Cost of $[A] \rightarrow [U] + [Y = b] = n^2 - n$

$\oplus Ux = Y = n^2$

Step	+ , -	* , /
1	$(n-k)(n-k)$	$(n-k)(n-k), (n-k)$
⋮		
k	$(n-k)^2$	$(n-k)^2, (n-k)$
⋮		
n-1		

$$\sum_{k=1}^{n-1} [2(n-k)^2 + (n-k)]$$

\Rightarrow let $t = n-k$
 $t=1 \Rightarrow$
 $t=n-1 \Rightarrow$

$$\begin{aligned} \sum_{t=1}^{n-1} 2(t)^2 + t &= 2 \sum_{t=1}^{n-1} t^2 + \sum_{t=1}^{n-1} t \\ &= 2 \frac{(n)(n-1)(2n-1)}{6} + \frac{n(n-1)}{2} \\ &= \frac{4n^3 - 3n^2 - n}{6} \end{aligned}$$

$$\begin{aligned} \text{Total Cost} &= \frac{4n^3 - 3n^2 - n}{6} + n^2 - n + n^2 \\ &= \frac{4n^3 + 9n^2 - 7n}{6} \approx \frac{2n^3}{3} \end{aligned}$$

* C.E

* نصف
 دایمی

Gauss Jordan Reduction

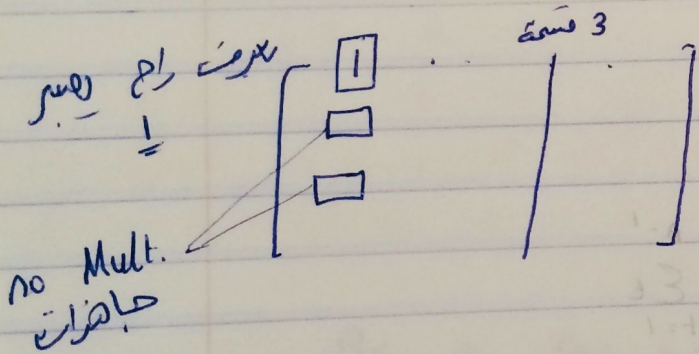
27-3-2018

$$[A \mid b] \rightarrow [I \mid x] \text{ for system } Ax = b$$

Ex 3x3

$$[A \mid b] = \left[\begin{array}{ccc|c} & & & \\ & & & \\ & & & \end{array} \right]$$

Step	+ , -	* , ÷
1	(2) 3	(2) 3 , 3 3-1+1
2	(2) (2)	2(2) , 2 3-2+1
3	2(1)	2(1) , 1
	12	12 , 6 $\Rightarrow 30$



n General for nxn

Step	+ , -	* , ÷
1		
2		
3		
⋮		
k	(n-1)(n-k+1)	(n-1)(n-k+1), (n-k+1)
⋮		
n		

Total Cost of GFR is

$$\sum_{k=1}^n [2(n-1)(n-k+1) + (n-k+1)]$$

$$= \frac{2n^3 + n^2 - n}{2} \approx n^3$$

* Inverse Method

because there is a unique solution

$$Ax = b$$

(1) Find A^{-1}

$$[A|I] \rightarrow [I|A^{-1}]$$

(2) $x = A^{-1} b_{n \times 1}$ Cost = $2n^2 - n$

Ex

$$\left[\begin{array}{ccc|ccc} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array} \right]_{3 \times 3}$$

Cost of $[A|I] \rightarrow [I|A^{-1}]$

Step	+	-	*	÷
1	2(5)		2(5)	5
2	2(4)		2(4)	4
3	2(3)		2(3)	3
	<u>24</u>		<u>24</u>	<u>12</u>

2 M
3 ⇒ قيمة المتجه الثاني

Cost of $A^{-1} = 60$

* Cost of $A^{-1}b = 2(3)^2 - 3 = 15$

Total Cost = $60 + 15 = 75$

For $n \times n$

Step	+, -	*, ÷
1		
2		
⋮		
k	$(n-1)(2n-k)$	$(n-1)(2n-k), (2n-k)$
⋮		
n		

Cost of A^{-1} is

$$\sum_{k=1}^n (2(n-1)(2n-k) + 2n-k)$$

$$\frac{6n^3 - 5n^2 + n}{2}$$

Total Cost of Inverse Method

$$\frac{6n^3 - 5n^2 + n}{2} + 2n^2 - n = \frac{6n^3 - n - n}{2} \approx 3n^3$$

↑
السريع

G.E $\frac{2}{3} n^3$

LU Fac $\frac{2}{3} n^3$

G-J-R n^3

Inverse $3n^3$

Cramer's $e(n+1)!$ Slowest

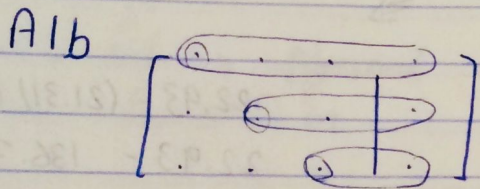
ليس مطلوب

السريع التالي

* Pivoting ^{نقطة} G.E [Finite-digit arithmetic]
 كل خطوة تقريب

$$Ax = b$$

$$A|b \rightarrow [u|c] + Bs$$



Pivoting: Switch the Pivotal row with a row below

Partial Pivoting \equiv Pivoting

نقطة
 أكبر واحد في الصف
 Pivot
 غير الصف
 Pivot

Ex Consider the system

$$1.133 x_1 + 5.251 x_2 = 6.414$$

$$24.14 x_1 - 1.210 x_2 = 22.93$$

Exact solution: $x_1 = 1, x_2 = 2$

① Solve this system using GE & 4 digit Rounding

② = = = = GE & = = = & Pivoting

Partial Pivoting

③ $[A|b] = \left[\begin{array}{cc|c} 1.133 & 5.281 & 6.414 \\ 24.14 & -1.210 & 22.93 \end{array} \right]$

$$M_{21} = \frac{24.14}{1.133} = 21.31$$

التقريب بعد
كل خطوة

$$\left[\begin{array}{cc|c} 1.133 & 5.281 & 6.414 \\ 0 & -113.7 & -113.8 \end{array} \right]$$

دورين حجاب

$$\begin{aligned} -1.210 & - (21.31)(5.281) \\ -1.210 & - 112.5 \\ \Rightarrow & -113.7 \end{aligned}$$

$$\begin{aligned} 22.93 & - (21.31)(6.414) \\ 22.93 & - 136.7 \\ \Rightarrow & -113.8 \end{aligned}$$

DS $X_2 = \frac{-113.8}{-113.7} = 1.001$

$$X_1 = \frac{6.414 - (5.281)(1.001)}{1.133}$$

$$= \frac{6.414 - 5.286}{1.133} = \frac{1.128}{1.133} = 0.9956$$

[2]

$$\left[\begin{array}{cc|c} 24.14 & -1.210 & 22.93 \\ 1.133 & 5.281 & 6.414 \end{array} \right]$$

بسطه كله
الوقت

$$M_{21} = \frac{1.133}{24.14} < 1$$

$$\boxed{X_1 = 1} \quad \boxed{X_2 = 1}$$