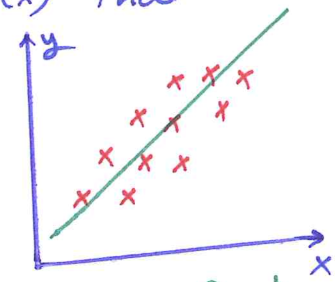
Ch 5 : Curve fitting

- Given a distinct points $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$
- Can we find a formula (or a curve) $y = f(x)$ that fits (or relates) these points?



- There are many different possibilities for the type of function that can be used.
- In this section we will study the class of linear function of the form: $y = f(x) = Ax + B$
- In ch4, we saw how to construct a polynomial that passes through a set of points. However, since f needs not to pass through these point, we can not use interpolation.
- Now we need to find A and B so we minimize the error (or deviation or residual): $|e_k| = |f(x_k) - y_k|$, $k = 1, 2, \dots, n$
- To handle the errors, we use norms to measure how far the curve $y = f(x)$ lies from the data.
- We consider the following norms:

① Maximum Error (ME): $E_\infty(f) = \text{Max } |e_k|$, $k \in \{1, 2, \dots, n\}$

[2] Average Error (AVE): $E_1(f) = \frac{1}{n} \sum_{k=1}^{n} |e_k|$

[3] Root-Mean-Square Error (RMSE): $E_2(f) = \sqrt{\frac{1}{n} \sum_{k=1}^{n} |e_k|^2}$

Exp Compare the ME, AVE, and the RMSE for the linear approximation $y = f(x) = 2x+1$ to the data points: $(1, 1.9)$, $(-1, -0.7)$, $(0, 1.2)$.

x_k	y_k	$f(x_k) = 2x_k + 1$	$\lvert e_k\rvert = \lvert f(x_k) - y_k\rvert$	$\lvert e_k\rvert^2$
1	1.9	3	1.1	1.21
-1	-0.7	-1	0.3	0.09
0	1.2	1	0.2	0.04

$$E_\infty(f) = \text{Max}\{1.1, 0.3, 0.2\} = 1.1$$

$$E_1(f) = \frac{1.1 + 0.3 + 0.2}{3} = \frac{1.6}{3} = 0.5\overline{3}$$

$$E_2(f) = \sqrt{\frac{1.21 + 0.09 + 0.04}{3}} = \sqrt{\frac{1.34}{3}} = \sqrt{0.44\overline{6}} \simeq 0.67$$

- Note that
  - E_∞ is the largest and if one point is badly in the error, its value determines E_∞.
  - E_1 averages the abs. value of the error. It is often used because it is easy to compute.
  - E_2 is the traditional choice because it is much easier to minimize.

Our task is to find a curve fitting that minimize E_2.