# Composite Trapezoidal Rule (CTR)



This method approximates the area under the curve $y = f(x)$ over $[a,b]$ using a series of trapezoids that lie above the intervals $\{[x_k, x_{k+1}]\}$.

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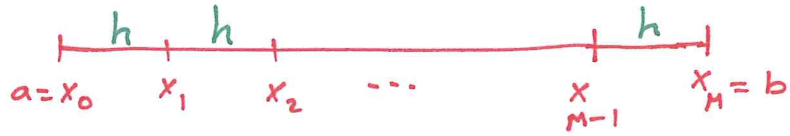
**Th** (CTR)

- Assume that the interval $[a,b]$ is subdivided into $M$ subintervals $[x_k, x_{k+1}]$ each of width $h = \frac{b-a}{M}$ using equally spaced nodes $x_k = a + kh$ for $k = 0, 1, 2, \dots, M$:




- Then, the composit trapezoidal rule is

$$\int_a^b f(x)\,dx = \int_{x_0}^{x_M} f(x)\,dx \approx T(f,h) = \int_{x_0}^{x_1} f(x)\,dx + \int_{x_1}^{x_2} f(x)\,dx + \cdots + \int_{x_{M-1}}^{x_M} f(x)\,dx$$

$$= \frac{h}{2}(f_0 + f_1) + \frac{h}{2}(f_1 + f_2) + \cdots + \frac{h}{2}(f_{M-1} + f_M)$$

$$= \frac{h}{2}\left(f_0 + 2f_1 + 2f_2 + \cdots + 2f_{M-1} + f_M\right)$$

- Furthermore, the total error of $T(f,h)$ is

$$E_T(f,h) = \frac{-h^3 f''(c)}{12} \cdot M = \frac{-h^3 f''(c)}{12} \cdot \frac{b-a}{h} = \frac{-h^2 f''(c)}{12}(b-a)$$

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Note that
- Number of points is $M+1$
- $M$ is the number of subintervals = number of subintegrals = number of composition

**Exp** Use CTR to estimate $\int_2^7 e^x\,dx$ with 10 composition. (use 4 chopping digits). 

- $h = \frac{b-a}{M} = \frac{7-2}{10} = \frac{5}{10} = 0.5$ and $f(x) = e^x = (2.718)^x$

| a | | | | | | | | | | b |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 | 5.5 | 6 | 6.5 | 7 |
| $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ |

(each subinterval has length $h$)

- $\int_2^7 e^x\,dx \simeq T(e^x, 0.5)$

$$= \frac{h}{2}\left[f_0 + 2f_1 + 2f_2 + 2f_3 + 2f_4 + 2f_5 + 2f_6 + 2f_7 + 2f_8 + 2f_9 + f_{10}\right]$$

$$= \frac{0.5}{2}\left[(2.718)^2 + 2(2.718)^{2.5} + 2(2.718)^3 + \cdots + 2(2.718)^{6.5} + (2.718)^7\right]$$

$$= 0.25\left[7.387 + 2(12.17) + 2(20.07) + \cdots + 2(664.6) + 1095\right]$$

$$= 0.25\left[7.387 + 24.34 + 40.14 + 66.2 + 109.1 + 179.9 + 296.6 + 489 + 806.2 + 1329 + 1095\right]$$

$$= 0.25\,(4333)$$

$$= 1083$$

True Vaule is $\int_2^7 e^x\,dx = e^x\Big|_2^7 = e^7 - e^2$

$= 1089.2441023295$

**Exp** Given

| $x$ | 0 | 2 | 4 | 6 |
|---|---|---|---|---|
| $f(x)$ | 10 | 15 | -10 | 8 |

Use CTR to estimate $\int_0^6 f(x)\,dx$

$$\int_0^6 f(x)\,dx \simeq \frac{h}{2}\left[f_0 + 2f_1 + 2f_2 + f_3\right] = \frac{2}{2}\left[10 + 2(15) + 2(-10) + 8\right] = 28$$

**Exp** Given

| $x$ | 0 | 2 | 3 | 6 |
|---|---|---|---|---|
| $f(x)$ | 10 | 15 | -10 | 8 |

Use CTR to estimate $\int_0^6 f(x)\,dx$.

$$\int_0^6 f(x)\,dx \simeq \int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx + \int_3^6 f(x)\,dx = \frac{2}{2}\left[f_0 + f_1\right] + \frac{1}{2}\left[f_1 + f_2\right] + \frac{3}{2}\left[f_2 + f_3\right]$$

$$= \left[10 + 15\right] + \frac{1}{2}\left[15 - 10\right] + \frac{3}{2}\left[-10 + 8\right] = 24.5$$