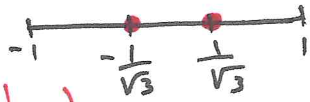
Gauss-Legendre Integration (optional)

To estimate the area under the curve $y = f(x)$, $-1 \le x \le 1 \Rightarrow$ one can use one of the following formulas:

1 Gauss-Legendre one-point Rule:

$$\int_{-1}^{1} f(x)\,dx \approx G_1(f) = 2 f_0$$

2 Gauss-Legendre two-points Rule:

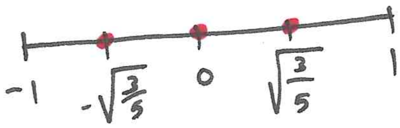


$$\int_{-1}^{1} f(x)\,dx \approx G_2(f) = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

Gauss-legendre two-points Rule $G_2(f)$ has Degree of Precision $\boxed{DP = 2n-1 = 3}$ and error $\boxed{E_2[f] = \frac{f^{(4)}(c)}{135}}$

3 Gauss-Legendre three-points Rule

$$\int_{-1}^{1} f(x)\,dx \approx G_3(f) = \frac{5 f\left(-\sqrt{\frac{3}{5}}\right) + 8 f(0) + 5 f\left(\sqrt{\frac{3}{5}}\right)}{9}$$



$G_3(f)$ has Degree of Precision $\boxed{DP = 2n-1 = 5}$

and error $\boxed{E_3[f] = \frac{f^{(6)}(c)}{15750}}$

Exp. Given $\int_{-1}^{1} \frac{dx}{x+2} = \ln(x+2)\Big|_{-1}^{1} = \ln(3) - \ln(1) \approx 1.09861$

- Estimate this integral using [1] $G_2(f)$ [2] $G_3(f)$ [3] $T(f,h)$ with $h=2$ [4] $S(f,h)$ with $h=1$

Use 4 chopping

[1] $\int_{-1}^{1} \frac{dx}{x+2} \approx G_2(f) = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$ $\qquad f(x) = \frac{1}{x+2}$

$= f(-0.5773) + f(0.5773)$

$= \frac{1}{1.422} + \frac{1}{2.577} = 0.7032 + 0.3880$

$= 1.091$

[2] $\int_{-1}^{1} \frac{dx}{x+2} \approx G_3(f) = \frac{5f\left(-\sqrt{\frac{3}{5}}\right) + 8f(0) + 5f\left(\sqrt{\frac{3}{5}}\right)}{9}$

$= \frac{5f(-0.7745) + 8f(0) + 5f(0.7745)}{9}$

$= \frac{4.081 + 4 + 1.802}{9} = \frac{9.883}{9} = 1.098$

[3] $\int_{-1}^{1} \frac{dx}{x+2} \approx T(f,2) = \frac{h}{2}\left[f(-1) + f(1)\right] = f(-1) + f(1)$

$= 1 + 0.3333 = 1.333$

[4] $\int_{-1}^{1} \frac{dx}{x+2} \approx S(f,1) = \frac{h}{3}\left[f(-1) + 4f(0) + f(1)\right]$

$= 0.3333\left[1 + 2 + 0.3333\right] = 0.3333(3.333) = 1.110$

Exp Estimate $\int_{-1}^{1} \sin x \, dx$ using [1] $G_2(f)$ [2] $G_3(f)$

1) $\int_{-1}^{1} \sin x \, dx \approx G_2(f) = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right) = -\sin\frac{1}{\sqrt{3}} + \sin\frac{1}{\sqrt{3}} = 0$

2) $\int_{-1}^{1} \sin x \, dx \approx G_3(f) = \frac{5f\left(-\sqrt{\frac{3}{5}}\right) + 8f(0) + 5f\left(\sqrt{\frac{3}{5}}\right)}{9} = 0$