Ch 9 : Numerical Approximation for IVP

- Given the IVP:

$$\frac{dy}{dt} = f(t, y(t)) \quad \text{with } y(t_0) = y_0$$

- To estimate the solution $y(t)$ numerically:
  - $\Rightarrow$ we find the values of y at different values of t
  - $\Rightarrow$ Then Approximate y using interpolation

Exp Consider the IVP: $y' = t + y \sin t^2$, $y(2) = -1$

Q4

- Clearly $t_0 = 2$ and $y_0 = -1$

- Write $y' = f(t, y)$
- Find $f'(t, y)$ by differentiating w.r.t t

$$f(t, y) = t + y \sin t^2$$

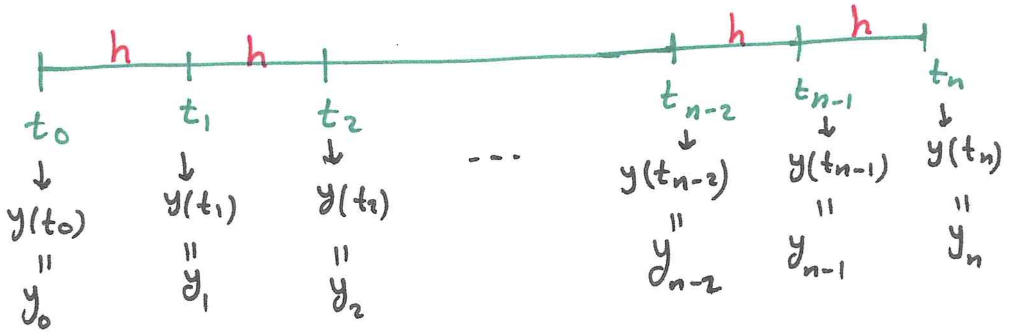
$$f'(t) = 1 + y\left[\cos t^2 \, (2t)\right] + \sin t^2 \, y'$$

$$= 1 + 2ty \cos t^2 + \sin t^2 \left(t + y \sin t^2\right)$$

General Principle:

- Given an IVP: $y' = f(t, y)$, $y(t_0) = y_0$
- To estimate the values of y on $[a, b] = [t_0, t_n]$:

$\Rightarrow$ Find $h = \dfrac{t_n - t_0}{n} = \dfrac{b-a}{n}$



$\Rightarrow$ Find $y(t_k) = y_k$ where $k = 0, 1, \dots, n$

and $t_k = t_0 + kh$

- We can find $y_1, y_2, \dots, y_n$ using

1. Euler's Method
2. Taylor's Method of order 2
3. Heun's Method
4. Runge-Kutta of order 4 (RK4)