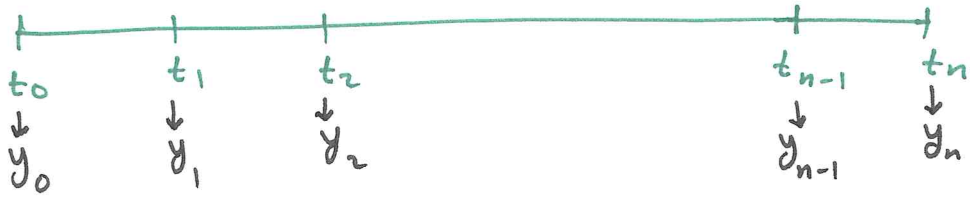
1 Euler's Method

- Given an IVP: $y' = f(t, y)$, $y(t_0) = y_0$
- $h = \dfrac{t_n - t_0}{n} = \dfrac{b-a}{n}$



- Take Taylor's expansion for $y(t)$ about $\underline{\underline{t_0}}$

$$y(t) \approx y(t_0) + (t - t_0)\, y'(t_0)$$
$$= y_0 + (t - t_0)\, f(t_0, y_0)$$
$$y(t_1) = y_0 + (t_1 - t_0)\, f(t_0, y_0)$$
$$\boxed{y_1 = y_0 + h\, f(t_0, y_0)}$$

- Similarly, take Taylor's expansion for $y(t)$ about $\underline{\underline{t_1}}$:

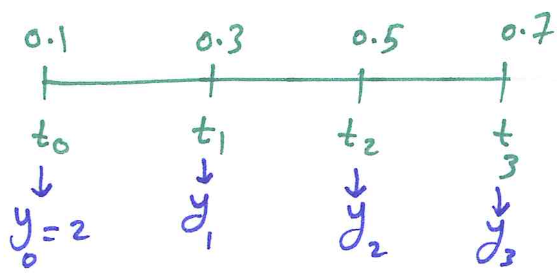
$$y(t) = y(t_1) + (t - t_1)\, y'(t_1)$$
$$y(t_2) = y_1 + (t_2 - t_1)\, f(t_1, y_1)$$
$$\boxed{y_2 = y_1 + h\, f(t_1, y_1)}$$
$$\vdots$$
$$\boxed{y_n = y_{n-1} + h\, f(t_{n-1}, y_{n-1})}$$

In general: The Euler's Method is

$$y_{k+1} \approx y_k + h\, f(t_k, y_k) \quad , \quad k = 0, 1, \ldots, n$$

Exp Given IVP: $y' = \dfrac{t-y}{2}$, $y(0.1) = 2$

Estimate $y(0.7)$ using Euler's Method with step size $h = 0.2$



$$f(t, y) = \frac{t - y}{2}$$

$$f(0.1, 2) = \frac{0.1 - 2}{2} = -0.95$$

$$y_1 = y_0 + h\, f(t_0, y_0)$$
$$= 2 + 0.2\, f(0.1, 2)$$
$$= 2 + 0.2(-0.95)$$
$$= 1.81$$

$$f(0.3, 1.81) = \frac{0.3 - 1.81}{2} = -0.755$$

$$y_2 = y(0.5) = y_1 + f(t_1, y_1)$$
$$= 1.81 + 0.2\, f(0.3, 1.81) = 1.81 + 0.2(-0.755)$$
$$= 1.659$$

$$y_3 = y(0.7) = y_2 + h\, f(t_2, y_2) = 1.659 + 0.2\, f(0.5, 1.659)$$
$$= 1.659 + 0.2\left(\frac{0.5 - 1.659}{2}\right)$$
$$= 1.5431$$