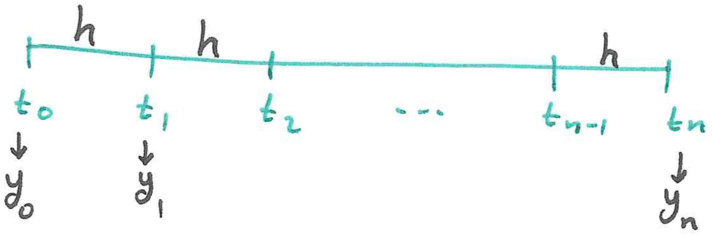
2 Taylor's Method of order 2

- Given an IVP: $y' = f(t,y)$, $y(t_0) = y_0$

  with $[a,b] = [t_0, t_n]$, $h = \dfrac{b-a}{n}$



- Take Taylor's expansion of order 2 for $y(t)$ about t_0 :

$$y \approx y(t_0) + (t-t_0)\, y'(t_0) + \frac{(t-t_0)^2}{2} y''(t_0)$$

$$\boxed{y_1 = y_0 + h\, f(t_0, y_0) + \frac{h^2}{2} f'(t_0, y_0)}$$

- Similarly: $y_2 = y_1 + h f(t_1, y_1) + \frac{h^2}{2} f'(t_1, y_1)$

$$\vdots$$

In general $\Rightarrow$

$$\boxed{y_{k+1} = y_k + h\, f(t_k, y_k) + \frac{h^2}{2} f'(t_k, y_k)}$$

for $k = 0, 1, 2, \dots, n-1$

Exp Given the IVP: $y' = \frac{t-y}{2}$, $y(1) = 2$

Estimate $y(1.4)$ using Taylor's Method of order 2 with step size $h = 0.4$

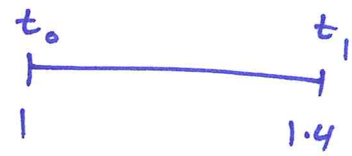
$$y_1 = y_0 + h f(t_0, y_0) + \frac{h^2}{2} f'(t_0, y_0)$$

$$= 2 + 0.4 f(1,2) + \frac{(0.4)^2}{2} f'(1,2)$$

$$= 2 + 0.4 \left(\frac{1-2}{2}\right) + 0.08 \left(\frac{1}{2} - \frac{1-2}{4}\right)$$

$$= 2 - 0.2 + 0.04 + 0.02$$

$$= 1.86$$



$t_0 = 1$, $y_0 = 2$

$f(t,y) = \frac{t-y}{2}$

$f'(t,y) = \frac{1}{2}(1 - y')$

$= \frac{1}{2} - \frac{t-y}{4}$