3 Heun's Method (Modification of Euler's Method)

- Given an IVP: $y' = f(t,y)$, $y(t_0) = y_0$
- Integrate both sides $\Rightarrow$

$$\int_{t_0}^{t_1} y'(t)\,dt = \int_{t_0}^{t_1} f(t,y)\,dt$$

$$y(t)\Big|_{t_0}^{t_1} = \frac{h}{2}\left[f(t_0,y_0) + f(t_1,y_1)\right]$$

$\Downarrow$ From Euler's Method

$$y(t_1) - y(t_0) = \frac{h}{2}\left[f(t_0,y_0) + f\left(t_1\,,\, y_0 + h f(t_0,y_0)\right)\right]$$

$$y_1 = y_0 + \frac{h}{2}\left[f(t_0,y_0) + f\left(t_1\,,\, y_0 + h f(t_0,y_0)\right)\right]$$

- Similarly $\Rightarrow$

$$y_2 = y_1 + \frac{h}{2}\left[f(t_1,y_1) + f\left(t_2\,,\, y_1 + h f(t_1,y_1)\right)\right]$$

$\vdots$

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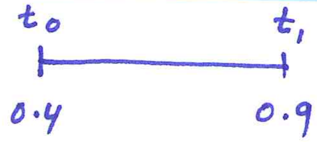
Exp Given the IVP: $t^2 + y' = (1+e^t)\,y^3$, $y(0.4) = 1$

Estimate $y(0.9)$ using Heun's Method with step size $h = 0.5$ and use 4 chopping digits.



- $h = 0.5$, $t_0 = 0.4$, $y_0 = 1$
- $f(t,y) = (1+e^t)\,y^3 - t^2$

$$= \left[1 + (2.718)^t\right] y^3 - t^2$$

- $f(t_0, y_0) = f(0.4, 1) = \left(1 + (2.718)^{0.4}\right)(1)^3 - (0.4)^2$

$$= 1 + 1.491 - 0.16$$
$$= 2.331$$

- $f(t_1, y_0 + h f(t_0, y_0)) = f\left(0.9, 1 + 0.5(2.331)\right) = f(0.9, 2.165)$

$$= \left[1 + (2.718)^{0.9}\right](2.165)^3 - (0.9)^2$$

$$= \left[1 + 2.459\right](10.14) - 0.81$$

$$= 35.07 - 0.81$$

$$= 34.26$$

- Hence, $y_1 = y(0.9) = y_0 + \frac{h}{2}\left[f(t_0, y_0) + f\left(t_1, y_0 + h f(t_0, y_0)\right)\right]$

$$= 1 + \frac{0.5}{2}\left[2.331 + 34.26\right]$$

$$= 1 + 0.25(36.59)$$

$$= 1 + 9.147$$

$$= 10.14$$