False Position Method - FPM
(Regula Falsi Method)

- Bisection method converges at slow speed
- However FPM converges faster.
- Conditions to solve $f(x)=0$: $f \in C[a,b]$ and
  $f(a)\,f(b) < 0$

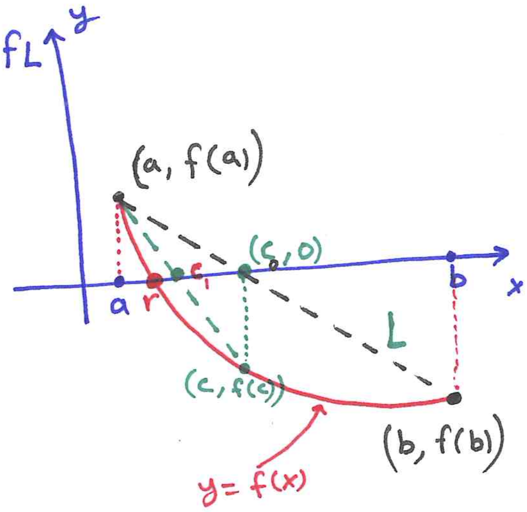
- The Bisection method uses the midpoint of $[a,b]$ as next iterate. A better approximation is to use the point $(c,0)$ where the secant L crosses the x-axis.

- To find c we equalize the slopes of L

$$\frac{f(b)-f(a)}{b-a} = \frac{0-f(b)}{c-b}$$

which gives the next iterat as

$$\boxed{c_0 = b_0 - \left(\frac{b_0 - a_0}{f(b_0)-f(a_0)}\right) f(b_0)}$$



- As in Bisection Method, we have three cases:
  1. If $f(a_0)\,f(c_0) < 0$, then the zero $r \in [a_0, c_0]$
  2. If $f(c_0)\,f(b_0) < 0$, then the zero $r \in [c_0, b_0]$
  3. If $f(c_0) = 0$, then the zero $r = c_0$.

- In general

$$c_n = b_n - \left(\frac{b_n - a_n}{f(b_n)-f(a_n)}\right) f(b_n)$$

$n = 0, 1, 2, 3, \dots$

Exp Use False Position Method to find the root of $x \sin x - 1 = 0$ that is located in the interval $[0,2]$

- $f(x) = x \sin x - 1$ with $[a_0, b_0] = [0,2]$
- $f(0) = -1 < 0$ and $f(2) = 0.81859485$

$$c_0 = b_0 - \left(\frac{b_0 - a_0}{f(b_0) - f(a_0)}\right) f(b_0) = 2 - \left(\frac{2-0}{0.81859485 - -1}\right) 0.81859485$$

$$= 1.09975017$$

- $f(c_0) = -0.02001921 \Rightarrow [a_1, b_1] = [c_0, b_0] = [1.0997507, 2]$

$$c_1 = b_1 - \left(\frac{b_1 - a_1}{f(b_1) - f(a_1)}\right) f(b_1) = 2 - \frac{(2 - 1.0997507)(0.81859485)}{0.81859485 - -0.02001921}$$

$$= 1.12124074$$

- $f(c_1) = 0.00983461 \Rightarrow [a_2, b_2] = [1.0997507, 1.12124074]$

⋮

n	a_n	c_n	b_n	$f(c_n)$
0	0	1.09975017	2	−0.02001921
1	1.09975017	1.12124074	2	0.00983461
2	1.09975017	1.11416120	1.12124074	0.00000563
3	1.09975017	1.11415714	1.11416120	$\boxed{0.00000000}$

To see the speed: $f(c_n) \to 0$ or $\{a_n - b_n\} \to 0$

Remark • The termination criterion in Bisection Method is not useful for the False Position method and may result in infinite loop.

This is in section 2.3

• Stopping Criterias (in general)

تفاوت

① $|f(c_n)| < \epsilon$ for given tolerance value ϵ given "mostly used"

② $|c_n - c_{n-1}| < \delta$ this is estimate for the absolute error

③ $2\frac{|c_n - c_{n-1}|}{|c_n| + |c_{n-1}|} < \delta$ this is estimate for the relative error