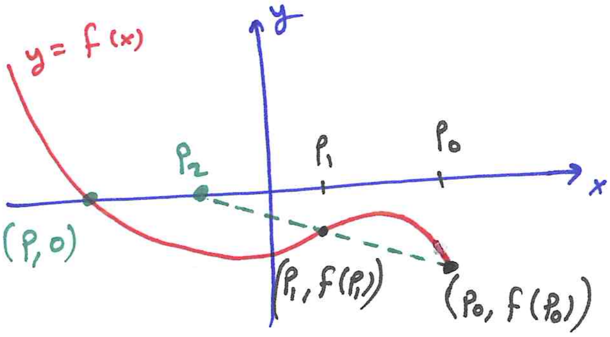
Secant Method

- Recall that in Newton-Raphson method, it is required the evaluation of $f(p_n)$ and $f'(p_n)$ per iteration since

$$p_{n+1} = p_n - \frac{f(p_n)}{f'(p_n)} \quad \text{for } n = 0, 1, 2, \dots$$

- It is desirable to have a method "secant method" that converges almost as fast as Newton's method and involves only evaluations of f and not f'.

$\Rightarrow$ Given $(p_0, f(p_0))$
$(p_1, f(p_1))$



$\Rightarrow$ To find p_2:

$$\frac{f(p_1) - f(p_0)}{p_1 - p_0} = m = \frac{0 - f(p_1)}{p_2 - p_1}$$

$\Rightarrow$ solve for p_2 $\Rightarrow$ $p_2 = p_1 - \left(\frac{p_1 - p_0}{f(p_1) - f(p_0)}\right) f(p_1)$

$\Rightarrow$ In general: $\boxed{p_{n+2} = p_{n+1} - \left(\frac{p_{n+1} - p_n}{f(p_{n+1}) - f(p_n)}\right) f(p_{n+1})}$

Exp Consider the equation $x = \cos x$. Take $p_0 = 0.5$ and $p_1 = \frac{\pi}{4}$ to find the next iteration "p_2" using secant method to approximate the solution of $x = \cos x$.

$3.14 \rightarrow \pi = 180$
$\frac{1}{2} \rightarrow ? = 28.7$

- $f(x) = x - \cos x$, $p_1 = \frac{\pi}{4} = 0.785$, $f(\frac{\pi}{4}) = \frac{\pi}{4} - \cos\frac{\pi}{4} = 0.785 - 0.707 = 0.078$

$f(\frac{1}{2}) = \frac{1}{2} - \cos(28.7) = {}^{-}0.377$

- $p_2 = p_1 - \left(\frac{p_1 - p_0}{f(p_1) - f(p_0)}\right) f(p_1) = \frac{\pi}{4} - \left(\frac{\frac{\pi}{4} - \frac{1}{2}}{f(\frac{\pi}{4}) - f(\frac{1}{2})}\right) f(\frac{\pi}{4}) = 0.73638414$