$\boxed{3}$ Seidel Iteration (Improvement of FPI)

- This method can be used to solve 2x2 or 3x3 non linear systems:

$$f_1(x,y)=0 \qquad\qquad f_1(x,y,z)=0$$
$$f_2(x,y)=0 \qquad \text{or} \qquad f_2(x,y,z)=0$$
$$f_3(x,y,z)=0$$

- For 2x2 system:

→ Write $x = g_1(x,y)$

$y = g_2(x,y)$

→ Given $(x_0, y_0) = (p_0, q_0)$

→ The Seidel Iteration is

$$p_{n+1} = g_1(p_n, q_n)$$

$$q_{n+1} = g_2(p_{n+1}, q_n)$$

- For 3x3 system:

→ Write $x = g_1(x,y,z)$

$y = g_2(x,y,z)$

$z = g_3(x,y,z)$

→ Given $(x_0, y_0, z_0) = (p_0, q_0, r_0)$

→ The Seidel Iteration is

$$p_{n+1} = g_1(p_n, q_n, r_n)$$

$$q_{n+1} = g_2(p_{n+1}, q_n, r_n)$$

$$r_{n+1} = g_3(p_{n+1}, q_{n+1}, r_n)$$

Exp • Consider the following non linear system:

$$x = e^x + zy$$
$$y = xy - x^2 z + 4$$
$$z = x^2 - yz$$

- Use Seidal iteration to find the next two approximations if the initial approximation of the solution is $(1, -1, 2)$.

- Use 4-significant digits.

$\boxed{87}$

- $x = g_1(x,y,z) = e^x + zy$

  $y = g_2(x,y,z) = xy - x^2z + 4$

  $z = g_3(x,y,z) = x^2 - yz$

- $(x_0, y_0, z_0) = (p_0, q_0, r_0) = (1,-1,2)$

- $p_1 = g_1(1,-1,2) = e - 2 = 0.7180$

  $q_1 = g_2(0.7180, -1, 2) = -0.7180 - (0.5155)(2) + 4 = 2.251$

  $r_1 = g_3(0.7180, 2.251, 2) = 0.5155 - 4.502 = -3.987$

- $p_2 = g_1(p_1, q_1, r_1) = g_1(0.7180, 2.251, -3.987) = -6.925$

  $q_2 = g_2(-6.925, 2.251, -3.987) = -202.8$

  $r_2 = g_3(-6.925, -202.8, -3.987) = -760.6$