$$Chapter 3$$

$$In this chapter we are goinig to solve n×n system of equations using the following methods:$$

1. $Newton^{'}s method:\left(2×2\right)system$

$$Need:1) f\_{1}(x,y)$$

$$ 2) f\_{2}(x,y)$$

$$ 3) initial guess (p\_{0},q\_{0})$$

$$Result: \left[\genfrac{}{}{0pt}{}{p\_{n+1}}{q\_{n+1}}\right]=\left[\genfrac{}{}{0pt}{}{p\_{n}}{q\_{n}}\right]-J^{-1}|\_{\left(p\_{n},q\_{n}\right)}\left[\genfrac{}{}{0pt}{}{f\_{1}\left(p\_{n},q\_{n}\right)}{f\_{2}\left(p\_{n},q\_{n}\right)}\right] ∴J=\left[\begin{matrix}\frac{∂f\_{1}}{∂x}&\frac{∂f\_{1}}{∂y}\\\frac{∂f\_{2}}{∂x}&\frac{∂f\_{2}}{∂y}\end{matrix}\right]$$

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1. $Fixed point iteration (FPI)$:

$$a) \left(2×2\right) system:$$

$$ Need:1) g\_{1}\left(x,y\right)=X$$

$$ 2) g\_{2}\left(x,y\right)=Y$$

$$ 3) initial guess (p\_{0},q\_{0})$$

$$ Result: \begin{matrix}p\_{n+1}= g\_{1}\left(p\_{n},q\_{n}\right)\\q\_{n+1}= g\_{2}\left(p\_{n},q\_{n}\right)\end{matrix}$$

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$$b) \left(3×3\right) system:$$

$$ Need:1) g\_{1}\left(x,y,z\right)=X$$

$$ 2) g\_{2}\left(x,y,z\right)=Y$$

$$ 3) g\_{3}\left(x,y,z\right)=Z$$

$$ 3) initial guess (p\_{0},q\_{0},r\_{0})$$

$$ Result: \begin{matrix}p\_{n+1}= g\_{1}\left(p\_{n},q\_{n},r\_{n}\right)\\\begin{matrix}q\_{n+1}= g\_{2}\left(p\_{n},q\_{n},r\_{n}\right)\\r\_{n+1}= g\_{3}\left(p\_{n},q\_{n},r\_{n}\right)\end{matrix}\end{matrix}$$

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1. $Gauss-seidel iteration:"speed up the FPI"$

$$a) \left(2×2\right) system:$$

$$ Need:1) g\_{1}\left(x,y\right)=X$$

$$ 2) g\_{2}\left(x,y\right)=Y$$

$$ 3) initial guess (p\_{0},q\_{0})$$

$$ Result: \begin{matrix}p\_{n+1}= g\_{1}\left(p\_{n},q\_{n}\right)\\ q\_{n+1}= g\_{2}\left(p\_{n+1},q\_{n}\right)\end{matrix}$$

$$Where:$$

$$\begin{matrix} p\_{1}= g\_{1}\left(p\_{0},q\_{0}\right)\\ q\_{1}= g\_{2}\left(p\_{1},q\_{0}\right)\end{matrix}$$

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$$b) \left(3×3\right) system:$$

$$ Need:1) g\_{1}\left(x,y,z\right)=X$$

$$ 2) g\_{2}\left(x,y,z\right)=Y$$

$$ 3) g\_{3}\left(x,y,z\right)=Z$$

$$ 3) initial guess (p\_{0},q\_{0},r\_{0})$$

$$ Result: \begin{matrix}p\_{n+1}= g\_{1}\left(p\_{n},q\_{n},r\_{n}\right)\\\begin{matrix} q\_{n+1}= g\_{2}\left(p\_{n+1},q\_{n},r\_{n}\right)\\ r\_{n+1}= g\_{3}\left(p\_{n+1},q\_{n+1},r\_{n}\right)\end{matrix}\end{matrix}$$

$$Where:$$

$$\begin{matrix} p\_{1}= g\_{1}\left(p\_{0},q\_{0},r\_{0}\right)\\ \begin{matrix}q\_{1}= g\_{2}\left(p\_{1},q\_{0},r\_{0}\right)\\r\_{1}= g\_{3}\left(p\_{1},q\_{1},r\_{0}\right)\end{matrix}\end{matrix}$$

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