$$Chapter 9$$

$$ solving initial value problems (IVP) or (differential equations)$$

$$\frac{dy}{dx}= y^{'}=f\left(t,y\left(t\right)\right) ; y\left(t\_{0}\right)=y\_{0}$$

$$We need to find y\_{0},y\_{1},y\_{2},y\_{3}, …$$

$$ – $Euler^{'}s Method:$

$$y\_{k+1}=y\_{k}+h\*f\left(t\_{k},y\_{k}\right)$$

$$Error= \frac{\left(b-a\right)}{2}\*h\*f^{''}\left(c\right)$$

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$$ – $Taylor Method of order 2:$

$$y\_{k+1}=y\_{k}+h\*f\left(t\_{k},y\_{k}\right)+\frac{h^{2}}{2}\*f^{'}(t\_{k},y\_{k})$$

$$Error= \frac{\left(b-a\right)}{6}\*h^{2}\*f^{'''}\left(c\right)$$

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$$ – $Heun's Method :$

$$y\_{k+1}=y\_{k}+\frac{h}{2}\left[f\left(t\_{k},y\_{k}\right)+f(t\_{k+1},\left(y\_{k}+h\*f\left(t\_{k},y\_{k}\right)\right)\right]$$

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$$ – $RK4 Method :$

$$y\_{k+1}=y\_{k}+\frac{h}{6}\left[f\_{1}+2f\_{2}+2f\_{3}+f\_{4}\right]$$

$$∴ f\_{1}=f\left(t\_{k},y\_{k}\right)$$

$$ f\_{2}=f\left(t\_{k}+\frac{h}{2} , y\_{k}+\frac{h}{2}f\_{1}\right)$$

$$ f\_{3}=f\left(t\_{k}+\frac{h}{2} , y\_{k}+\frac{h}{2}f\_{2}\right)$$

$$ f\_{4}=f\left(t\_{k}+h , y\_{k}+h\*f\_{3}\right)$$

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