Math330

Assignment #2 (Part 2)

First semester 2019/2020 — Chapter 2 — Mahmoud Ghannam

(Q1) Consider the function $h(x) = 0.001x^4 + x^2 + 3x + 1$. Estimate the critical point of $h(x)$ using Newton's method with a guess value of -1 and accuracy of 5×10^{-5}.

(Q2) Calculate an approximate value for $4^{\frac{1}{4}}$ using two steps of the secant method starting with $P_0 = 1$ and $P_1 = 2$

(Q3) Let $f(x) = (x-1)^2 \ln x$. Use the accelerated Newton's method to estimate the zero of $f(x)$ with $p_0 = 1.5$ and $|E| < 5 \times 10^{-7}$

(Q4) A small business has weekly profits of $P(x) = x^2 + 4x - 2e^x$, where x is the number of units produced weekly. Using Newton's method with $p_0 = 1.5$ and $\epsilon = 10^{-6}$, approximate the level of production that yields the maximum profit.

(Q5) Clearly $P = \pi$ is a root of $f(x) = (\pi - x)\sin(x - \pi)$.

(a) Find the multiplicity of this root.

(b) Find the order of convergence and the asymptotic error constant if we used Newton-Raphson iteration to estimate this root.

(c) Use the accelerated Newton's method to estimate this root using $P_0 = 3$. Find two iterations.

(d) What is the order of convergence and the asymptotic error constant for bisection method to estimate this root?

(e) What is the order of convergence of secant method to estimate this root?

(f) What is the order of convergence of false-position method to estimate this root?

(Q6) Let $P = 2$ be a fixed point of $g(x)$ with $g'(2) = g''(2) = 0$ and $g'''(2) = -4.8$, find R and A of the FPI.

(Q7) Clearly $p = 1$ ia root of $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$. Find R and A if we used Newton's method. Then prove your results numerically using $p_0 = 1.5$ with four iterations.

(Q8) Clearly $p = 3$ ia root of $f(x) = \ln(x-2) + 3x - 9$. Find R and A of the following methods:

(a) Newton-Raphson method.

(b) The secant method.

(Q9) Consider the fixed point iteration: $p_{n+1} = g(p_n) = \dfrac{p_n(p_n^2 + 6)}{3p_n^2 + 2}$

(a) Show that $p = \sqrt{2}$ is a fixed point of $g(x)$.

(b) Find the order of convergence and asymptotic error constant.