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Math 330
Assignment #3

Q.1:- $f(x,y) = e^x - \ln(2x+y) + 1$
$g(x,y) = x^2y + \cos x + \frac{2}{y}$

$(P_0, q_0) = (0,1)$, find (P_1, q_1) By N.M :-

$$\Rightarrow \begin{pmatrix} P_1 \\ q_1 \end{pmatrix} = \begin{pmatrix} P_0 \\ q_0 \end{pmatrix} - J^{-1}(P_0,q_0) \begin{pmatrix} f(P_0,q_0) \\ g(P_0,q_0) \end{pmatrix} \text{ --- } (*)$$

$$f(P_0,q_0) = f(0,1) = e^0 - \ln(2(0)+1) + 1 = 1 - 0 + 1 = 2$$
$$g(P_0,q_0) = g(0,1) = (0) + \cos(0) + \frac{2}{(1)} = 0 + 1 + 2 = 3$$

$$J(x,y) = \begin{pmatrix} f_x & f_y \\ g_x & g_y \end{pmatrix} = \begin{pmatrix} e^x - \frac{2}{2x+y} & \frac{-1}{2x+y} \\ 2xy - \sin x & x^2 - \frac{2}{y^2} \end{pmatrix}$$

$$J(P_0,q_0) = \begin{pmatrix} e^0 - \frac{2}{1} & \frac{-1}{1} \\ 0 - \sin 0 & 0 - \frac{2}{1} \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ 0 & -2 \end{pmatrix}$$

$$J^{-1}(P_0,q_0) = \frac{1}{2}\begin{pmatrix} -2 & 1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0.5 \\ 0 & -0.5 \end{pmatrix}$$

* In $(*)$:-

$$\begin{pmatrix} P_1 \\ q_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 & 0.5 \\ 0 & -0.5 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 + 3(0.5) \\ 0 + -0.5(3) \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} -0.5 \\ -1.5 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} P_1 \\ q_1 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 2.5 \end{pmatrix}$$

Q.2:-

$x = x^2 + 3y + e^z$

$y = 2x + 3y$ $\qquad (p_0, q_0, r_0) = (0.1, 0.2, 0)$

$z = 9x - y^2 + \cos z$

$\boxed{1}$ F.P.I :-

- $p_1 = g_1(p_0, q_0, r_0) = g_1(0.1, 0.2, 0) = (0.1)^2 + 3(0.2) + e^0 = 1.61$ $\quad p_1$
- $q_1 = g_2(p_0, q_0, r_0) = g_2(0.1, 0.2, 0) = 2(0.1) + 3(0.2) = 0.8$ $\quad q_1$
- $r_1 = g_3(p_0, q_0, r_0) = g_3(0.1, 0.2, 0) = 9(0.1) - (0.2)^2 + \cos 0 = 1.86$ $\quad r_1$

$\Rightarrow (p_1, q_1, r_1) = (1.61, 0.8, 1.86)$

$\boxed{2}$ Seidel Iteration :-

- $p_1 = g_1(p_0, q_0, r_0) = g_1(0.1, 0.2, 0) = (0.1)^2 + 3(0.2) + e^0 = 1.61$
- $q_1 = g_2(p_1, q_0, r_0) = g_2(1.61, 0.2, 0) = 2(1.61) + 3(0.2) = 3.82$
- $r_1 = g_3(p_1, q_1, r_0) = g_3(1.61, 3.82, 0) = 9(1.61) - (3.82)^2 + \cos 0 = 0.8976$

$\Rightarrow (p_1, q_1, r_1) = (1.61, 3.82, 0.8976)$

Q.3:- A, B and C 4x4 matrices,

The cst $2A + B - |C|C^3$

+ $|C|$ requires cost = 63

$[n!e - 2] = [4!e - 2] = 63.$

+ C^3 requires cost = 224

$C^3 = \underset{(2n^3 - n^2)}{C^2} \cdot C \qquad \frac{C^2 \cdot C}{(2n^3 - n^2)}$

$C^3 = 2(2n^3 - n^2) = 2(2(4)^3 - (4)^2) = 224$

+ $|C| \cdot C^3$ requires cost = 16

* $2A$ requires cost = 16

* $+B$ requires cost = 16

* $\ominus C|C|^3$ requires cost = 16

Total cost = $63 + 224 + 16 + 16 + 16 + 16$
$= 351$

Q.7:– a] $A = \begin{pmatrix} 2 & 1 & 2 \\ 4 & 3 & -18 \\ -8 & 1 & 1 \end{pmatrix}$ $b = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$

$A = LU$

Step 1:– $m_{21} = \frac{a_{21}}{a_{11}} = \frac{4}{2} = 2 \Rightarrow R_2 - 2R_1$

$m_{31} = \frac{a_{31}}{a_{11}} = \frac{-8}{2} = -4 \Rightarrow R_3 + 4R_1$

$$\begin{pmatrix} 2 & 1 & 2 \\ 0 & 1 & -5 \\ 0 & 5 & 9 \end{pmatrix}$$

cost : $\div$: 2
$+,-$: 4 $\Rightarrow$ 10
$\times$: 4

Step 2:– $m_{32} = \frac{a_{32}}{a_{22}} = \frac{5}{1} = 5 \Rightarrow R_3 - 5R_2$

$$U = \begin{pmatrix} 2 & 1 & 2 \\ 0 & 1 & -5 \\ 0 & 0 & 34 \end{pmatrix}$$

cost = $\div$: 1
$*,-$: 1 $\Rightarrow$ 3
$\times$: 1

$$L = \begin{pmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -4 & 5 & 1 \end{pmatrix}$$

cost LU
$\Downarrow$
$10 + 3$
$\Downarrow$
13

* solve $LY = b$:-

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 2 & 1 & 0 & 1 \\ -4 & 5 & 1 & 3 \end{array}\right) \Rightarrow \begin{array}{l} y_1 = 5 \\ y_2 = 1 - 2(5) = -9 \\ y_3 = 3 - 5(-9) + 4(5) = 68 \end{array}$$

cost : $\div$: 0
$+,-$: 3
$\times$: 3
6

* Now solve $UX = Y$

$$\left(\begin{array}{ccc|c} 2 & 1 & 2 & 5 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 34 & 68 \end{array}\right) \Rightarrow \begin{array}{l} X_3 = \frac{68}{34} = 2 \\ X_2 = -9 + 5(2) = 1 \\ X_1 = 5 - 2(2) - 1(1) = 0 \end{array}$$

cost : $\div$: 3
$+,-$: 3
$\times$: 3
9

$\Rightarrow X_1 = 0,\ X_2 = 1,\ X_3 = 2.$

b) cost :- cost of $A = LU$:- 13

" " F.S :- 6 $\Rightarrow \boxed{28}$

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" " B.S :- 9

or total cost = [cost of F.S + cost of B.S + cost of $A = LU$]

$$= (n^2 - n) + (n^2) + \frac{4n^3 - 3n^2 - n}{6}$$

$$= \frac{6n^2 - 6n - 6n^2 + 4n^3 - 3n^2 - n}{6} = \left.\frac{9n^2 - 7n + 4n^3}{6}\right|_{n=3} = \boxed{28}$$

c) Total cost of LU factorization for $n \times n$ system :-

cost of F.S $= n^2 - n$.

cost of B.S $= n^2$

cost of $A = LU = \frac{4n^3 - 3n^2 - n}{6}$

$$\text{total cost} = n^2 - n + n^2 + \frac{4n^3 - 3n^2 - n}{6}$$

$$= \frac{6n^2 - 6n + 6n^2 + 4n^3 - 3n^2 - n}{6}$$

$$= \frac{4n^3 + 9n^2 - 7n}{6}$$