Math330

Assignment #5

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(Q1) Consider the function $f(x) = \frac{3}{x} - x$ with the nodes 1 and 3 respectively.

(a) Find the natural cubic spline based on these nodes, then use it to estimate $f(2)$.

(b) Find the clamped cubic spline based on these nodes, then use it to estimate $f(\frac{7}{3})$.

(Q2) A clamped cubic spline $S(x)$ for a function $f(x)$ is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + Bx + 2x^2 - 2x^3 \ ; & 0 \le x \le 1 \\ S_1(x) = 1 + b(x-1) - 4(x-1)^2 + 7(x-1)^3 \ ; & 1 \le x \le 2 \end{cases}$$

Find $f'(0)$ and $f'(2)$. Then estimate $f(0.4)$ and $f(1.5)$

(Q3) A natural cubic spline $S(x)$ is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + B(x-1) - D(x-1)^3 \ ; & 1 \le x \le 2 \\ S_1(x) = 1 + b(x-2) - \frac{3}{4}(x-2)^2 + d(x-2)^3 \ ; & 2 \le x \le 3 \end{cases}$$

If $S(x)$ interpolates the data $(1,1), (2,1),$ and $(3,0)$, Find $B, D, b,$ and d

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(Q4) Given the data

x	1	2	4	5
y	2	8	4	6

(a) Find the least-squares line fits the data. Then find the root-mean-square error.

(b) Find the least-squares fit of the form $f(x) = Ax^2 + Bx$. Then estimate $f(2.6)$.

(c) Find the normal equations of the fitting curve of the form $y = \sin(Ax) + B\ln(x)$.

(d) Use linearization to find the curve fitting of the form $y = axe^{bx}$.

(e) Use linearization to find the fit of the form $g(x) = \frac{Cx}{D+x}$. Then estimate y when $x = 3$

(Q5) Given the data:

x	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
y	-1	0	1	2	1

If we want to fit the data with least-square curve $f(x) = A\sin(\pi x) + \frac{1}{3}\cos(\pi x)$, find A

(Q6) Consider the points: $(1.1, 1.6622), (1.2, 2.4562), (1.3, 3.3943)$ with the fitting curve $f(x) = Ax^3 + B\cos x$

(a) Find the normal equations of the fitting curve.

(b) Use the normal equations to find A and B

(c) Use linearization to find A and B
