Math330

Assignment #6

2nd semester 2018/2019 — Mahmoud Ghannam

(Q1) The following table represents the displacement d of an object after t seconds.

t	1	1.2	1.4	1.6
d	2	3	4	5

Approximate the object's velocity v at $t = 1.4$ seconds using the following formulas (if possible)

(a) The central difference formula of order $O(h^2)$
(b) The central difference formula of order $O(h^4)$
(c) The forward difference formula of order $O(h^2)$
(d) The backward difference formula of order $O(h^2)$

(Q2) Derive the following formula using: (a) Newton's interpolation. (b) Lagrange's interpolation. (c) Taylor's expansion.

$$f''(x_0) = \frac{3f_{-1} - 4f_0 + f_3}{6h^2} - \frac{2hf^{(3)}(c)}{3}$$

(Q3) Use the nodes $x_0 - 4h$ and $x_0 + h$ to derive $f'(x_0)$ with its truncation error using:

(a) Newton's interpolation. (b) Lagrange's interpolation. (c) Taylor's expansion.

(Q4) Consider the data:

x	2.3	2.6	2.9	3.2
$f(x)$	3	-9	A	B

If $f'(2.9) \approx 95$ using the backward difference formula of order $O(h^2)$, and $f'(2.9) \approx 40$ using the central difference formula of order $O(h^2)$, find the values of A and B.

(Q5) Consider the function $f(x) = \sqrt[3]{x+7}$ with $1 \le x \le 1.5$. Find the optimal h for the formula below assuming that $\epsilon = 5 \times 10^{-9}$.

$$f'''(x_0) = \frac{-f_0 + 3f_1 - 3f_2 + f_3}{h^3} - \frac{9hf^{(4)}(c)}{8}$$

(Q6) Consider the difference formula: $f'(x_0) = \frac{4f_2 - 3f_0 - f_{-4}}{12h} - \frac{4h^2 f^{(3)}(c)}{3}$

(a) Derive the formula using Taylor's series.

(b) Derive the formula using Newton's interpolation.

(c) Derive the formula using Lagrange's interpolation.

(d) Find the optimal h.

(e) Use the formula with the points $(4, 1.246), (4.2, 1.870), (4.4, 2.600), (4.6, 3.259)$ to estimate $f'(4.4)$.
