Math330

Assignment #7

2nd semester 2018/2019 — Mahmoud Ghannam

(Q1) Approximate the integral $\int_0^2 e^{-x}\, dx$ using:

(a) Trapezoidal rule.
(b) Trapezoidal rule with four compositions.
(c) Simpson's rule.
(d) Simpson's rule with two compositions.
(e) Simpson's 3/8 rule.
(f) Gauss-Legendre one-point rule.
(g) Gauss-Legendre two-point rule.
(h) Gauss-Legendre three-point rule.
(i) How many compositions M do we need to estimate the above integral with accuracy of 5×10^{-9} using:
(1) Composite Trapezoidal rule. (2) Composite Simpson's rule.

(Q2) Estimate the value of $\int_0^{14} f(x)dx$ using composite Simpson's rule using the data:
$(0, 19), (1, 13), (2, 10), (4, 17), (6, 5), (10, 2)$, and $(14, 1)$.

(Q3) The velocity of a body is given by $v(t) = \begin{cases} 2t & ; \quad 1 \le t \le 5 \\ 5t^2 + 3 & ; \quad 5 < t \le 14 \end{cases}$

Find the distance covered by the body between t=2 and t=8 using two-composition trapezoidal rule

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(Q4) Consider the quadrature formula: $\int_{-h}^{h} f(x)dx \approx Q[f] = \frac{h}{2}(f(-h) + 3f(\frac{h}{3}))$.

Find the degree of precision and $E[f]$ of $Q[f]$.

(Q5) Consider the quadrature formula: $\int_{-1}^{1} f(x)dx \approx Q[f] = Af(\frac{-1}{3}) + Bf(\frac{1}{3}) + Cf(1)$

Assuming that the degree of precision of $Q[f]$ is 2,

(a) Determine the constants A, B, and C.

(b) Find the truncation error of $Q[f]$.

(Q6) If the average value of $f(x)$ on $[a, b]$ is given by $A = \frac{1}{b-a}\int_a^b f(x)\, dx$, Estimate A using trapezoidal rule.

(Q7) Estimate $\int_a^{a+2h} (x-a)^4\, dx$ using Gauss-Legendre two-point rule

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