Birzeit University
Mathematics Department
Second Semester 2012 / 2013
MATH330 – First Exam

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Question 1 (8 points)

Given the function

$$g(x) = \frac{x^2 - \cos(x)}{5}$$

a) Using **Fixed Point Theorems** show that $g(x)$ has a fixed point in the interval $[-1,1]$.
b) Using **Fixed Point Theorems** show that the fixed point in the interval $[-1,1]$ is unique.
c) Using part b) and $p_0 = 1$ what is the **theoretical number of iterations** needed by Fixed Point Iteration to get an error less than or equal to 10^{-3} .
d) Using $p_0 = 1$ write the results obtained by Fixed Point Iteration to get an error less than 10^{-3} .
e) How many iterations were done in part d) ?

a) ~~$g(x) \in C[a,b]$~~

~~$g(1) = \frac{1-\cos(1)}{5} = 0.0919395$~~

~~$g(-1) = \frac{1-\cos(-1)}{5} = 0.0919395$~~

$g(0) = \frac{0-0}{5} = 0$

$-1 \le x \le 1$ $\cos(x) \le$

* maximum value for $\cos(x) = 1$
* ~~minimum value for~~ $\cos(x) = -1$

$1 \ge \cos(x) \ge -1$

$-1 \le -\cos(x) \le 1$

$x^2 - 1 \le x^2 - \cos(x) \le x^2 + 1$

$\frac{x^2-1}{5} \le \frac{x^2-\cos(x)}{5} \le \frac{x^2+1}{5}$

so $\frac{x^2+1}{5} \ge g(x) \ge \frac{x^2-1}{5}$

$\frac{2}{5} \ge g(x) \ge 0$ for $x \in [-1,1]$

$\Rightarrow$ range$(g) \subseteq [-1,1]$ $\Rightarrow \exists$ a fixed point

$g'(x) = \frac{2x + \sin x}{5}$ * $\sin x \le 1$

so $g'(x)$ is maximum $\frac{2x+1}{5}$

خلف الورقة →

Question 2 (4 points)

Given the function

$$f(x) = e^x + x - 4$$

a) Calculate c_1 using **Bisection Method** on the interval [0,2].
b) Calculate c_1 using **False Position Method** on the interval [0,2].

a) $f(0) = e^0 + 0 - 4 = -3$

$f(2) = e^2 + 2 - 4 = 5.389056099$

$c_0 = \frac{b_0 + a_0}{2} \rightarrow \frac{2+0}{2} = 1$

$f(1) = e + 1 - 4 = -0.281718171$

$c_1 = \frac{b_1 - a_1}{2} = \frac{2+1}{2} = \frac{3}{2} = 1.5$

b) $c_0 = b_0 - \frac{f(b_0)(b_0 - a_0)}{f(b_0) - f(a_0)}$

$= 2 - \frac{f(2)(2)}{f(2) - f(0)} \rightarrow 2 - \frac{2f(2)}{f(2)+3}$

$\rightarrow c_0 = 0.715217532$

$f(c_0) = ?$

$\Rightarrow c_1 = b_0 - \frac{f(b_0)(b_0 - c_0)}{f(b_0) - f(c_0)}$

$c_1 = 2 - \frac{f(2)(2 - 0.715217232)}{f(2) - f(0.715217532)}$

~~1.044433~~ $c_1 = 0.955566687$

3.5

Question 3 (4 points)

A plane is taking off and its altitude (x) in meters after (t) seconds is given by the following function

$$x(t) = e^{\frac{t}{2}} - e^{-\frac{t}{2}} - t$$

After how many seconds is the altitude 70 meters? **Stop iterating when the successive error is less than or equal to 0.001 seconds.**

$70 = e^{\frac{t}{2}} - e^{-\frac{t}{2}} - t$

$f(x) = e^{\frac{t}{2}} - e^{-\frac{t}{2}} - t - 70 = 0$

$\Rightarrow e^{\frac{t}{2}} - e^{-\frac{t}{2}} - t - 70 = 0 \Rightarrow f(x)$

let $P_0 = 1$

using Newton method

$P_{n+1} = P_n - \frac{f(x)}{f'(x)}$

$= P_0 - \frac{e^{\frac{1}{2}t} - e^{-\frac{1}{2}t} - t - 70}{\frac{1}{2}e^{\frac{1}{2}t} + \frac{1}{2}e^{-\frac{1}{2}t} - 1}$

-69.649591761

.128

65.12454535

$1.9196881[illegible] \times 10^{28}$

$9.5[illegible] \times 10^{38}$

69.4578093[illegible]

0.127625465

P_1

$P_2 =$

$P_3 =$

$P_4 =$

Question 4 (4 points)

a) A matrix A of size $N \times N$ is constructed using the following pseudo-code

```
for i = 1 to N
    for j = 1 to N
        A_{i,j} = 1/(i+j-1) - 1/i - 1/j + 2
    end
end
```

Find the cost of evaluating the matrix A.

b) A matrix B of size 3×3 is constructed using the following pseudo-code

```
for i = 1 to 3
    for j = 1 to 3
        B_{i,j} = 1/(i+j-1) + 1
    end
end
```

Use 4-digit chopping to evaluate the matrix B.

for each element
Aij we have 5 Additions & subtractions
& 3 Multiplications
so for each element Aij the cost is 5+3 = (8)
and since ~~we~~ we have N*N matrix containing
N^2 elements
the total cost is $8N^2$ ✓

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Question 5 (5 points)

Given the system

$$e^x + e^y - 4 = xy$$

$$\sin(x) + \cos(y-4) = 0$$

use $(p_0, q_0) = (1,0)$ and Newton's method to evaluate (p_1, q_1).

$f_1 = e^x + e^y - 4 - xy$

$f_2 = \sin(x) + \cos(y-4)$

$J \Delta P = -F$

$J = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{pmatrix} = \begin{pmatrix} e^x - y & e^y - x \\ \cos(x) & -\sin(y-4) \end{pmatrix}$

at $(p_0, q_0) = (1,0)$ $\Big|$ $J = \begin{pmatrix} e & 0 \\ \cos(1) & -0.756802495 \end{pmatrix}$

$f_1 = e^x + e^y - 4 - xy \rightarrow f_1 = e + 1 - 4 - 0 = -0.28171817$

$f_2 = \sin(x) + \cos(y-4) = 0 \rightarrow f_2 = \sin(1) + \cos(-4) = 0.18782363$

$\begin{pmatrix} 0.28171817 \\ -0.18782363 \end{pmatrix} = \begin{pmatrix} e & 0 \\ 0.540302305 & -0.756802495 \end{pmatrix} \begin{pmatrix} \Delta p \\ \Delta q \end{pmatrix}$

$e\Delta p = 0.28171817$

$\Delta p = 0.103638323$

$p_1 - 1 = 0.103638323 \Rightarrow p_1 = \boxed{1.103638323}$

~~Δq = 0.540302305 Δp~~

$0.540302305\, \Delta p - 0.756802495\, \Delta q = -0.18782363$

$= 0.055996024 - 0.756802495\, \Delta q = -0.18782363$

$q_1 = \boxed{0.322175...}$