Birzeit University
Mathematics Department
Second Semester 2013/2014
MATH 330 – Exam No. 1
Instructor: Dr. Hani Kabajah

Name (بالعربية): Key Form 2 Student No.: __________

Sections: Put a tick in the last column

Section	Time		Your section
1	T, R	09:30 – 11:00	
2	S, M, W	09:00 – 10:00	
3	S, M, W	10:00 – 11:00	

Question 1 (20 points)

Circle the correct answer then fill the following table.

In the table write A, B, C, or D in the answer column.

Question	Answer
1	B
2	D
3	B
4	B
5	B
6	C
7	B
8	B
9	A
10	B

Question	Answer
11	C
12	C
13	A
14	B
15	C
16	C
17	C
18	D
19	B
20	C

1) Let $f(x) = x^3 - 10x + 4$, using Newton method and $p_0 = -2$ then p_7 equals

a) -3.3496327
b) -3.345969012
c) -3.345963296
d) None of the above.

2) Let $f(x) = x^3 - 10x + 4$, using bisection method on the interval $[0, 3]$ then c_1 equals

a) 1.5
b) 2.25
c) 0.75
d) None of the above.

3) Let $f(x) = x^3 - 10x + 4$, using false position method on the interval $[0, 2]$ then c_1 equals

a) 0.6667
b) 0.4186
c) 0.9148
d) 1.1111

4) Let $f(x) = x^3 - 10x + 4$, using secant method and $p_0 = 0$ and $p_1 = 1$ then p_2 equals

a) 0.6667
b) 0.4444
c) 1.5556
d) 0.9148

5) The interpolation polynomial of the following data points (0,0), (1,1), (2,3), (3,−1), (4,4) is of degree

a) At least 4
b) At most 4
c) At most 5
d) At least 5

6) Polynomial Interpolation of the data points (x_0, y_0), (x_1, y_1), …, (x_n, y_n) means
   a) Finding a function $f(x)$ such that $f(x) = y$ for all x
   b) Finding a polynomial $P_n(x)$ such that $P_n(x) = y$ for all x
   c) Finding a polynomial $P_n(x)$ such that $P_n(x_k) = y_k$ for all x_k, $k = 0,1, \dots, n$
   d) None of the above.

7) Using interpolation by polynomial of the following data points (0,0), (1,1), (2,3) the value of $f(1.5)$ is best approximated by
   a) 2
   b) 1.875
   c) 2.25
   d) 2.375

8) The function $g(x) = \frac{4}{x^2} + 2$ has a fixed point in the interval [1,10].
   Which statement is true?
   a) Using the Fixed Point Theorem we are sure that the function $g(x)$ has a unique fixed point in the interval [1,10].
   b) Using the Fixed Point Theorem we cannot know if the function $g(x)$ has a unique fixed point in the interval [1,10].
   c) Using the Fixed Point Theorem we are sure that the Fixed Point Iteration of the function $g(x)$ converges for any $p_0 \in [1,10]$.
   d) None of the above.

9) Let $f(x) = (x-1) \cdot \sin(\pi x)$. If we use the secant method to find the root $p = 1$ then the order of convergence will be
   a) $R = 1$
   b) $R = \frac{1+\sqrt{5}}{2}$
   c) $R = 2$
   d) None of the above.

10) The system

$$\begin{aligned} y &= x^3 \\ x^2 + y^2 &= 1 \end{aligned}$$

   a) has one solution
   b) has two solutions
   c) has three solutions
   d) has four solutions

11) Using 7-digit rounding the value of $f(x) = \ln(1+x^2)$ at $x = 10^{-4}$ is

a) 9.999999×10^{-9}
b) 1
c) 0
d) 1×10^{-8}

12) Given the following two approximations

$$\begin{aligned} f(h) &= 1 + h^3 + O(h^5) \\ g(h) &= h + h^2 + O(h^3) \end{aligned}$$

The value of $f(0.1) + g(0.1)$ is best approximated by

a) 1.1
b) 1.11
c) 1.111
d) None of the above

13) The function $f(x) = \ln(1+x^2) - 4$ has a root in the interval

a) $[7, 8]$
b) $[-7, -6]$
c) $[7, 8]$ and $[-7, -6]$
d) None of the above

14) If we solve the following system using Gaussian elimination, partial pivoting, and 3-digit chopping

$$\begin{aligned} 0.121\,x + 0.287\,y &= 0.399 \\ 1.12\,x + 2.87\,y &= 3.99 \end{aligned}$$

The solution is

a) $(x, y) = (0.00892, 1.39)$
b) $(x, y) = (-0.0178, 1.40)$
c) $(x, y) = (-0.0178, 1.39)$
d) $(x, y) = (0.00892, 1.38)$

15) To approximate $\sin(2.01)$ with smallest error we shall use

a) $2.01 - \frac{(2.01)^3}{3!}$
b) $2.01 - \frac{(2.01)^3}{3!} + \frac{(2.01)^5}{5!}$
c) $2 - \frac{(2)^3}{3!} + \frac{(2)^5}{5!} - \frac{(2)^7}{7!}$
d) None of the above

16) A linear system $AX = B$ is called ill-conditioned if

a) $\det(A) \approx 0$
b) Large changes in A makes large changes in X
c) Small changes in A makes large changes in X
d) None of the above

17) To solve the system

$$\begin{aligned} x\sin(y) + y\sin(x) &= x \\ e^x - e^y &= y \end{aligned}$$

Using Seidel method and $(p_0, q_0) = (1, 1)$ then

a) $(p_1, q_1) = (0.0349, -1.683)$
b) $(p_1, q_1) = (1.682, 4.376)$
c) $(p_1, q_1) = (1.682, 2.658)$
d) $(p_1, q_1) = (1.682, 0)$

18) Let A be a 3×3 matrix. The cost calculating A^2 then solving $A^2X = B$ (using Gaussian Elimination and Back Substitution) is

a) 63
b) 72
c) 42
d) 73

19) Find the distance between the points (4,2,3) and (5,3,6) using $L4$ norm.

a) 3.072
b) 3.018
c) 3.317
d) 3

20) One of these is not a stopping criteria for solving $x = g(x)$

a) $|p_{n+1} - p_n| < \delta$

b) $\frac{|p_{n+1} - p_n|}{|p_n|} < \delta$

c) $\frac{2|p_{n+1} - p_n|}{|p_{n+1}| - |p_n|} < \delta$

d) $\frac{2|p_{n+1} - p_n|}{|p_{n+1}| + |p_n|} < \delta$

Question 2 (5 points)

Given the system

$$\begin{aligned} x^2 + y^2 - 25 &= 0 \\ x^2 - y - 3 &= 0 \end{aligned}$$

Use Newton method and $(p_0, q_0) = (3,4)$ to calculate (p_1, q_1).

Q2

$$x^2+y^2-25=0$$
$$x^2-y-3=0$$

$$(p_0,q_0)=(3,4) \quad , \quad (p_1,q_1)=\,?$$

$$\begin{pmatrix} p_1 \\ q_1 \end{pmatrix} = \begin{pmatrix} p_0 \\ q_0 \end{pmatrix} - \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{pmatrix}^{-1} \begin{pmatrix} f_1(p_0,q_0) \\ f_2(p_0,q_0) \end{pmatrix}$$

$$\frac{\partial f_1}{\partial x}=2x \qquad \frac{\partial f_1}{\partial y}=2y$$

$$\frac{\partial f_2}{\partial x}=2x \qquad \frac{\partial f_2}{\partial y}=-1$$

$$J(p_0,q_0)=\begin{pmatrix} 2p_0 & 2q_0 \\ 2p_0 & -1 \end{pmatrix} = \begin{pmatrix} 6 & 8 \\ 6 & -1 \end{pmatrix}$$

$$J^{-1}(p_0,q_0)=\begin{pmatrix} 6 & 8 \\ 6 & -1 \end{pmatrix}^{-1} = \frac{1}{-54}\begin{pmatrix} -1 & -8 \\ -6 & 6 \end{pmatrix}$$

$$\begin{pmatrix} p_1 \\ q_1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \left(\frac{1}{-54}\begin{pmatrix} -1 & -8 \\ -6 & 6 \end{pmatrix}\right)\begin{pmatrix} f_1(3,4) \\ f_2(3,4) \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \frac{1}{54}\begin{pmatrix} -1 & -8 \\ -6 & 6 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \frac{1}{54}\begin{pmatrix} -16 \\ 12 \end{pmatrix} = \begin{pmatrix} 3-16/54 \\ 4+12/54 \end{pmatrix}$$

$$= \begin{pmatrix} 2\frac{19}{27} \\ 4\frac{2}{9} \end{pmatrix} = \begin{pmatrix} 2.704 \\ 4.222 \end{pmatrix}$$