key

Name...........................   Number................   Section ....1...........

(Q1) [*20 points*] Consider the function $f(x) = x + \frac{3}{x}$ with the points: $(1, 4), (1.25, 3.65), (1.5, 3.5)$

(a) Estimate $f(1.2)$ using Newton's polynomial.

(b) Find $L_{2,2}(1.4)$ with its cost.

(c) Find the upper bound of the interpolation error.

(a) $P_2(x) = a_0 + a_1(x-1) + a_2(x-1)(x-1.25)$ ③

x_k	y_k	1st D.D	2nd D.D
1	4 ①	—	—
1.25	3.65	-1.4 ①	—
1.5	3.5	-0.6	1.6 ①

$P_2(x) = 4 - 1.4(x-1) + 1.6(x-1)(x-1.25)$ ②

$f(1.2) \approx 4 - 1.4(1.2-1) + 1.6(1.2-1)(1.2-1.25) = \boxed{3.704}$

(b) $L_{2,2}(x) = \dfrac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \dfrac{(x-1)(x-1.25)}{(1.5-1)(1.5-1.25)}$ ②

$L_{2,2}(1.4) = \dfrac{(1.4-1)(1.4-1.25)}{(1.5-1)(1.5-1.25)} = \boxed{0.48}$ ②

$Cost = 7$ ⑧

$\Rightarrow$

(C) $\quad |E_2(x)| \leq \dfrac{h^3 M_3}{9\sqrt{3}}$ ①

$$h = 0.25 \quad \text{①}$$

$$f = x + \frac{3}{x}$$

$$f' = 1 - \frac{3}{x^2}$$

$$f'' = \frac{6}{x^3}$$

$$f''' = \frac{-18}{x^4}$$

$$|f'''(x)| = \frac{18}{x^4} \leq \frac{18}{1^4} = \boxed{18 = M_3} \quad \text{②}$$

$$\Rightarrow \quad |E_2(x)| \leq \frac{(0.25)^3 \, 18}{9\sqrt{3}} = \underline{0.0180421 95}$$

②

key

(Q1) [*20 points*] Consider the function $f(x) = x + \frac{3}{x}$ with the points: $(1, 4), (1.25, 3.65), (1.5, 3.5)$

(a) Estimate $f(1.4)$ using Lagrange's polynomial.

(b) Find $f[1, 1.25, 1.5]$ with its cost.

(c) Find the upper bound of the interpolation error.

(a) $P_2(x) = y_0 \dfrac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + y_1 \dfrac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + y_2 \dfrac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$ ③

$f(1.4) \approx P_2(1.4)$.

$= 4 \dfrac{(1.4-1.25)(1.4-1.5)}{(1-1.25)(1-1.5)} + 3.65 \dfrac{(1.4-1)(1.4-1.5)}{(1.25-1)(1.25-1.5)}$ ③

$+ 3.5 \dfrac{(1.4-1)(1.4-1.25)}{(1.5-1)(1.5-1.25)}$

$= \boxed{3.5360}$ ②

(b) $f[1, 1.25, 1.5] = \dfrac{\dfrac{f(1.5)-f(1.25)}{1.5-1.25} - \dfrac{f(1.25)-f(1)}{1.25-1}}{1.5-1}$ ②

$= \boxed{1.6}$ ② $\boxed{Cost = 9}$ ②

OR

x_k	y_k	1st D.D	2nd D.D
1	4	—	—
1.25	3.65	−1.4	—
1.5	3.5	−0.6	$\boxed{1.6}$

1

(C) $\left| E_2(x) \right| \leq \dfrac{h^3 M_3}{9\sqrt{3}}$ ①
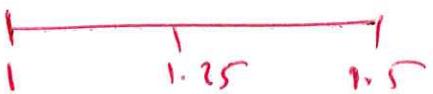
$h = 0.25$ ①

$f' = 1 - \dfrac{3}{x^2}$

$f'' = \dfrac{6}{x^3}$

$f''' = \dfrac{-18}{x^4}$

$\left| f'''(x) \right| = \dfrac{18}{x^4} \leq \dfrac{18}{1} = \boxed{18 = M_3}$ ②

$\left| E_2(x) \right| \leq \dfrac{(0.25)^3 \, 18}{9\sqrt{3}}$

$\left| E_2(x) \right| \leq 0.01804295$ ②