

key

Quiz # 5

Math330

2018/2019

Name.....

Number.....

Section 5

Q1) [20 points] Given the points (1, -1), (2, 3), (5, 24.6)

(a) Find the least-squares fit of the form $y = Ax^2 + \frac{B}{x}$, then use it to estimate $f(4)$

(b) Find a suitable linearization for the fitting curve $y = Ax^2 + \frac{B}{x}$

$$(a) E(A, B) = \sum_{k=1}^n \left(Ax_k^2 + \frac{B}{x_k} - y_k \right)^2$$

$$\frac{\partial E}{\partial A} = 0 \Rightarrow \sum 2 \left(Ax_k^2 + \frac{B}{x_k} - y_k \right) \cdot 2x_k = 0$$

$$A \sum x_k^4 + B \sum x_k = \sum x_k^2 y_k \quad \text{--- (1)}$$

$$\frac{\partial E}{\partial B} = 0 \Rightarrow \sum 2 \left(Ax_k^2 + \frac{B}{x_k} - y_k \right) \cdot \frac{1}{x_k} = 0$$

$$\Rightarrow A \sum x_k + B \sum \frac{1}{x_k^2} = \sum \frac{y_k}{x_k} \quad \text{--- (2)}$$

x_k	y_k	x_k^4	$x_k^2 y_k$	$\frac{1}{x_k^2}$	$\frac{y_k}{x_k}$
1	-1	1	-1	1	-1
2	3	16	12	0.25	1.5
5	24.6	625	615	0.04	4.92
8	26.6	642	626	1.29	5.42

$$642A + 8B = 626$$

$$8A + 1.29B = 5.42$$

\Rightarrow

$$A = \frac{\begin{vmatrix} 626 & 8 \\ 5.42 & 1.29 \end{vmatrix}}{\begin{vmatrix} 642 & 8 \\ 8 & 1.29 \end{vmatrix}} = \frac{764.18}{764.18} = 1$$

$$B = \frac{\begin{vmatrix} 642 & 626 \\ 8 & 5.42 \end{vmatrix}}{\begin{vmatrix} 642 & 8 \\ 8 & 1.29 \end{vmatrix}} = \frac{-1528.36}{764.18} = -2$$

$$\Rightarrow y = x^2 - \frac{2}{x} \Rightarrow f(x) \approx x^2 - \frac{2}{x} = 15.5$$

$$(b) \quad y = Ax^2 + \frac{B}{x}$$

$$\Rightarrow \boxed{xy} = A \boxed{x^3} + B$$

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