Birzeit University
Mathematics Department

key

Quiz # 5 Math330 2018/2019

Name..... Number..... Section ..... 1 .....

Q1) [*20 points*] Given the points $(1.1, 2.5667), (1.2, 3.2737), (1.3, 4.1316)$

(a) Find the normal equations of the best fit of the form $f(x) = Ax^3 + B\ln x$

(b) Use linearization to find the best fit of the form $f(x) = Ax^3 + B\ln x$ (use four decimal digits)

(a) $E(A,B) = \sum (Ax_k^3 + B\ln x_k - y_k)^2$ (1)

$\frac{\partial E}{\partial A} = \sum 2(Ax_k^3 + B\ln x_k - y_k)\cdot x_k^3 = 0$ (2)

$\frac{\partial E}{\partial B} = \sum 2(Ax_k^3 + B\ln x_k - y_k)\cdot \ln x_k = 0$ (2)

(b) $y = Ax^3 + B\ln x \Rightarrow \boxed{\frac{y}{\ln x}} = A\boxed{\frac{x^3}{\ln x}} + B$

Y (2) $\quad X$ (2)

x_k	y_k	X_k	Y_k	X_k^2	X_kY_k
1.1	2.5667	13.9649	26.9300	195.0184	376.0748
1.2	3.2737	9.4778	17.9556	89.8287	170.1796
1.3	4.1316	8.3739	15.7476	70.1222	131.8689
		31.8166	60.6332	354.9693	678.1233
		(1)	(1)	(1)	(1)

$$A\sum x_k^2 + B\sum x_k = \sum x_k y_k$$

$$A\sum x_k + nB = \sum y_k$$

$$\Rightarrow \begin{cases} 354.9693A + 31.8166B = 678.1233 \\ 31.8166A + 3B = 60.6332 \end{cases} \quad (4)$$

$$A = \frac{\begin{vmatrix} 678.1233 & 31.8166 \\ 60.6332 & 3 \end{vmatrix}}{\begin{vmatrix} 354.9693 & 31.8166 \\ 31.8166 & 3 \end{vmatrix}} = \frac{105.2276}{52.6119} \approx 2 \quad (2)$$

$$B = \frac{\begin{vmatrix} 354.9693 & 678.1233 \\ 31.8166 & 60.6332 \end{vmatrix}}{\begin{vmatrix} 354.9693 & 31.8166 \\ 31.8166 & 3 \end{vmatrix}} = \frac{-52.6532}{52.6119} \approx -1 \quad (2)$$

$$\boxed{f(x) = 2x^3 - \ln x}$$