20

key

Birzeit University
Mathematics Department

Quiz # 2 Math330 2018/2019

Name................................ Number................. Section ....1............

Q1) [*20 points*] Let $g(x) = e^{-x} + \frac{1}{x} + 2$.

(1) Show that $g(x)$ has a fixed point p in the interval $I = [2, 3]$.

(2) Show that if $p_0 \in I$, then the fixed point iteration will converge to p.

(3) Estimate p starting with $p_0 = 2.5$ (Do only four iterations)

(4) Find the number of iterations needed to estimate p with accuracy of 5×10^{-10}

(1) g is decreasing 2

$\Rightarrow \quad g(3) \le g(x) \le g(2)$

$2 < 2.38 \le g(x) \le 2.64 < 3$ 2

Range $\subseteq$ Domain

$\Rightarrow$ Existence 2

(2) $|g'(x)| = \left| -e^{-x} - \frac{1}{x^2} \right| = \left| e^{-x} + \frac{1}{x^2} \right| = e^{-x} + \frac{1}{x^2}$

$|g'(x)|$ is decreasing 2

$|g'(x)| \le e^{-2} + \frac{1}{2^2} = \boxed{0.385335283} < 1$ 1

k

3

$\Rightarrow$ uniqueness & Convergence

(3) $p_0 = 2.5$

$p_1 = 2.482084999$

$p_2 = 2.486455892$

$p_3 = 2.485383191$

$p_4 = 2.485646074$

4

(4) $n > \dfrac{\ln\left(\dfrac{\epsilon(1-k)}{|p_1 - p_0|}\right)}{\ln k}$ 2

$n > 18.8$ 1

$\boxed{n \geq 19}$ 1

20

key

Birzeit University
Mathematics Department

Quiz # 2 Math330 2018/2019

Name.................................. Number................. Section ......5..........

Q1) [*20 points*] Let $g(x) = \sqrt[4]{x+1}$.

(1) Show that $g(x)$ has a fixed point p in the interval $I = [1,2]$.

(2) Show that if $p_0 \in I$, then the fixed point iteration will converge to p.

(3) Estimate p starting with $p_0 = 1.5$ (Do only four iterations)

(4) Find the number of iterations needed to estimate p with accuracy of 5×10^{-10}

Sol: (1) g is increasing 2

$\Rightarrow$ $g(1) \le g(x) \le g(2)$

$1 < 1.189 \le g(x) \le 1.316 < 2$ } 2

$\Rightarrow$ Range $\subseteq$ Domain $\Rightarrow$ Existence.

(2) $g'(x) = \frac{1}{4}(x+1)^{-3/4} = \frac{1}{4\sqrt[4]{(x+1)^3}}$ 2

$|g'(x)| = \frac{1}{4\sqrt[4]{(x+1)^3}}$ which is decreasing 2

$\Rightarrow |g'(x)| \le \frac{1}{4\sqrt[4]{(1+1)^3}} = \underbrace{\boxed{0.148650889}}_{k} < 1$ 3 1

$\Rightarrow$ Convergence & uniqueness.

$\Longrightarrow$

(3) $p_0 = 1.5$

$p_1 = 1.257433347$

$p_2 = 1.225755182$

$p_3 = 1.221432153$

$p_4 = 1.220838632$

4

(4) $$n > \frac{\ln\left(\frac{\epsilon(1-k)}{|p_1 - p_0|}\right)}{\ln k}$$ 2

$n > 10.576$ 1

$n \geq 11$ 1