BIRZEIT UNIVERSITY

MATHEMATICS DEPARTMENT
MATH330

• Name..... Key ..... • Number..... • Section.....6.....

Q_1. Use $(p_0, q_0) = (0.1, -2)$ to estimate the solution of the following system using Gauss Seidel iteration:

$$x = x + y - xy + 2, \quad = g_1(x, y)$$
$$y = xe^{-y} - 1. \quad = g_2(x, y)$$

Use 4 significant digits rounded to find the first 2 iterations.

$p_1 = g_1(0.1, -2) = 0.1 - 2 + 0.2 + 2 = \boxed{0.3}$

$q_1 = g_2(0.3, -2) = 0.3\,e^{2} - 1 = 0.3(2.718)^2 - 1 = 0.3(7.388) - 1$
$= 2.216 - 1 = \boxed{1.216}$

$p_2 = g_1((0.3), (1.216)) = 0.3 + 1.216 - (0.3)(1.216) + 2 =$
$= 1.516 - 0.3648 + 2 = 1.151 + 2 = \boxed{3.151}$

$q_2 = g_2(3.151, 1.216) = 3.151\,(2.718)^{-1.216} - 1 = 3.151(0.2965) - 1$
$= 0.9343 - 1 = \boxed{-0.0657}$

Q_2. Find the Jacobian matrix for the following system at $(p_0, q_0) = (0.1, -2)$.

$$f_1(x, y) = x + y - xy + 2$$
$$f_2(x, y) = xe^{-y} - 1.$$

$$J = \begin{pmatrix} 1 - y & 1 - x \\ e^{-y} & -xe^{-y} \end{pmatrix} \Rightarrow J\Big|_{(0.1, -2)} = \begin{pmatrix} 3 & 0.9 \\ 7.3891 & -0.73891 \end{pmatrix}$$

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• Name..... Key ..... • Number..... • Section.....3.....

Q_1. Use $(p_0, q_0) = (0.1, -2)$ to estimate the solution of the following system using Newton Method:

$$f_1(x, y) = x + y - xy + 2 = 0,$$

$$f_2(x, y) = xe^{-y} - 1 = 0.$$

Use 4 significant digits rounded to find (p_1, q_1).

$f_1(0.1, -2) = 0.3$, $f_2(0.1, -2) = -0.2611$

$$J = \begin{pmatrix} 1-y & 1-x \\ e^{-y} & -xe^{-y} \end{pmatrix}\Bigg|_{(0.1,-2)} = \begin{pmatrix} 3 & 0.9 \\ 7.389 & -0.7389 \end{pmatrix}$$

$$J^{-1} = \frac{1}{-8.867}\begin{pmatrix} -0.7389 & -0.9 \\ -7.389 & 3 \end{pmatrix}$$

$$\begin{pmatrix} p_1 \\ q_1 \end{pmatrix} = \begin{pmatrix} 0.1 \\ -2 \end{pmatrix} - \frac{1}{-8.867}\begin{pmatrix} -0.7389 & -0.9 \\ -7.389 & 3 \end{pmatrix}\begin{pmatrix} 0.3 \\ -0.2611 \end{pmatrix}$$

$$= \begin{pmatrix} 0.1 \\ -2 \end{pmatrix} + \frac{1}{8.867}\begin{pmatrix} 0.0133 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} 0.1 \\ -2 \end{pmatrix} + \begin{pmatrix} 1.5 \times 10^{-3} \\ -0.3383 \end{pmatrix} = \begin{pmatrix} 0.1015 \\ -2.338 \end{pmatrix}$$