BIRZEIT UNIVERSITY

MATHEMATICS DEPARTMENT
MATH330

• Name..... key ..... • Number..... • Section.....3.....

Q_1. Consider the function $f(x) = 1 + \dfrac{1}{(x+1)^2}$.

a) Estimate $f(2.5)$ using Newton polynomial with the nodes $(1, f(1)), (2, f(2)), (3, f(3))$.

b) Find the upper bound for the interpolation error.

a) $P_2(x) = a_0 + a_1(x-1) + a_2(x-1)(x-2)$

x_k	$f[x_k]$	$f[x_{k-1}, x_k]$	$f[x_{k-2}, x_{k-1}, x_k]$
1	1.25	—	—
2	1.1111	−0.1389	—
3	1.0625	−0.0486	0.04515

$P_2(x) = 1.25 + -0.1389(x-1) + 0.04515(x-1)(x-2)$

$P_2(2.5) = 1.25 - 0.2084 + 0.03386$

$= 1.07546.$

b. $|E_2(x)| \le \dfrac{h^3 M_3}{9\sqrt{3}}$, $M_3 = \max_{[1,3]} \left| \dfrac{-24}{(1+x)^5} \right|$

$|E_2(x)| \le \dfrac{(1)^3(0.75)}{9\sqrt{3}} = 0.048113.$

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• Name.....Key..... • Number..... • Section.....6.....

Q_1. Consider the function $f(x) = 1 + \dfrac{1}{(x+1)^2}$.

a) Estimate $f(2.5)$ using Lagrange interpolation with the nodes $(1, f(1)), (2, f(2)), (3.5, f(3.5))$.

b) Find the upper bound for the interpolation error when estimating $f(2.5)$.

(a) $f(1) = 1.25$, $f(2) = 1.1111$, $f(3.5) = 1.0494$

$$P_2(x) = \frac{(x-2)(x-3.5)}{(1-2)(1-3.5)}(1.25) + \frac{(x-1)(x-3.5)}{(2-1)(2-3.5)}(1.1111) + \frac{(x-1)(x-2)}{(3.5-1)(3.5-2)}(1.0494)$$

$$P_2(2.5) = \frac{(2.5-2)(2.5-3.5)}{(1-2)(1-3.5)}(1.25) + \frac{(2.5-1)(2.5-3.5)}{(2-1)(2-3.5)}(1.1111) + \frac{(2.5-1)(2.5-2)}{(3.5-1)(3.5-2)}(1.0494)$$

$$\Rightarrow P_2(2.5) = 1.070975$$

(b) $$E_2(x) = \frac{(x-1)(x-2)(x-3.5)\,f^{(3)}(c)}{3!}$$

$$|E_2(2.5)| \le \left|\frac{(2.5-1)(2.5-2)(2.5-3.5)\,M_3}{3!}\right|, \quad M_3 = \max_{[1,3.5]}\left|\frac{-24}{(1+x)^5}\right|$$

$$|E_2(2.5)| \le \frac{|(1.5)(0.5)(-1)|(0.75)}{3!} = 0.09375$$