Q3

BIRZEIT UNIVERSITY

MATHEMATICS DEPARTMENT
MATH330

• Name.......Key................ • Number................... • Section.........3..........

(10 points)

Q_1. Consider the equation $f(x) = (x-1)\sin \pi x$.
Find the order of convergence and the asymptotic error constant both theoretically and numerically.

Theoritically

$P=1$ is a root of multiplicity $\boxed{2}$ $\Rightarrow$ Multiple root

since $f(1) = 0 = f'(1)$ & $f''(1) \neq 0$.

$\Rightarrow$ $R=1$ & $A = \frac{2-1}{2} = \frac{1}{2}$

Numerically: Take $P_0 = 0.9$. then

$f'(x) = \pi(x-1)\cos \pi x + \sin \pi x$.

$P_{n+1} = P_n - \frac{f(P_n)}{f'(P_n)}$

	P_n	$\lvert P - P_n \rvert$	$\frac{\lvert E_{n+1} \rvert}{\lvert E_n \rvert}$
P_0	0.9	0.1	—
P_1	0.95084	0.04916	0.4916
P_2	0.975519	0.024481	0.49798
P_3	0.98777	0.01223	0.49957
P_4	0.993887		
	0.99694		
	0.99847		≈ 0.5
	0.99921		

Q2

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• Name......Key.............. • Number.................... • Section.........6..........

Q_1. Consider the function $g(x) = \sqrt{\sin x + 1}$.
a) Show that $g(x)$ has a fixed point in the interval $[0, \frac{\pi}{2}]$.
b) Show that the fixed point iterations of $g(x)$ converges for any $p_0 \in [0, \frac{\pi}{2}]$.
c) Use $p_0 = 1.5$, find number of iterations needed to get accuracy of 5×10^{-2}.

a) • $g(x)$ is cont. on $[0, \frac{\pi}{2}]$

• g is Increasing on $[0, \frac{\pi}{2}]$

$g(0) = 1$

$g(\frac{\pi}{2}) = 1.4$

$\Rightarrow g(x) \in [0, \frac{\pi}{2}]$

$\Rightarrow$ $\exists$ at least one fixed point.

b) $|g'(x)| = \dfrac{|\cos x|}{2\sqrt{1+\sin x}}$ which is decreasing func. on $[0, \frac{\pi}{2}]$

$\Rightarrow g'(0) = \dfrac{1}{2\cdot 1} = \dfrac{1}{2}$ & $g'(\frac{\pi}{2}) = 0$

$\Rightarrow K = \frac{1}{2} < 1 \Rightarrow$ The Iteration Converges.

c) $P_0 = 1.5$, $P_1 = g(1.5) = 1.413333$

$\Rightarrow \dfrac{(\frac{1}{2})^n \, |P_1 - P_0|}{1 - 0.5} < 5 \times 10^{-2}$

$(\frac{1}{2})^n < 0.28845 \Rightarrow n \ln \frac{1}{2} < \ln 0.28845$

$n > 1.79 \Rightarrow \boxed{n \approx 2}$

Q2

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MATH330

• Name....Key.................... • Number.................... • Section.........3..........

Q_1. Consider the function $g(x) = 1 + \frac{1}{(x+1)^2}$. (10 points)

a) Show that $g(x)$ has a fixed point in the interval $[1, 2]$.

b) Show that the fixed point iterations of $g(x)$ converges for any $p_0 \in [1, 2]$.

c) Use $p_0 = 1.5$, find number of iterations needed to get accuracy of 5×10^{-8}.

a) • g is cont. on $[1,2]$

• g is decreasing on $[1,2]$

3) $g(1) = \frac{5}{4} = 1.25 \in [1,2]$

$g(2) = 1.1111 \in [1,2]$

$\Rightarrow g(x) \in [1,2] \ \forall x \in [1,2]$

then there exist at least one fixed point in $[1,2]$.

b) $g'(x) = \frac{-2}{(1+x)^3}$. which is increasing function on $[1,2]$ & $|g'(x)|$ is decreasing

4) $g'(1) = -0.25$

$g'(2) = -0.07407$

$\Rightarrow k = \max_{[1,2]} |g'(x)| = 0.25$

Since $k < 1$, then for any $p_0 \in [1,2]$, F.P.I converges

c) $P_0 = 1.5$, $P_1 = 1.16 \Rightarrow \frac{(0.25)^n |1.16 - 1.5|}{1 - 0.25} < 5 \times 10^{-8}$

3) $\Rightarrow (0.25)^n < 1.10294 \times 10^{-7} \Rightarrow n > 11.5561 \Rightarrow \boxed{n = 12}$

Q1

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• Name....Key.... • Number.................... • Section.........3..........

Q_1. Assume we approximate $\sin x$ by $x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$. Find an upper bound for the error when estimating $\sin 0.4$.

$$\text{upper bound} \leq \max_{x\in(0,0.4)} \frac{|f^{(8)}(x)|\, x^8}{8!} \quad , \quad |f^{(8)}(x)| = \sin x \text{ which is Increasing on } (0, 0.4)$$

$$\leq \frac{\sin(0.4)\,(0.4)^8}{8!}$$

$$= \frac{2.55209 \times 10^{-4}}{40320} \approx 6.32959 \times 10^{-9}$$

Q_2. Use Bisection method to estimate the root of $\cos x - \ln x + 1 = 0$ on the interval $[1, 2]$. Find 4 iterations.

a_n	c_n	b_n	$f(c_n)$
1^+	1.5	2^-	+0.66527
1.5^+	1.75	2^-	0.262138
1.75^+	1.875	2^-	0.071857
1.875	$\boxed{1.9375}$	2^-	

Q1

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• Name.....Key..... • Number.................. • Section.........6..........

Q_1. Assume we approximate $\cos x$ by $1 - \frac{x^2}{2} + \frac{x^4}{24}$. Find an upper bound for the error when estimating $\cos 0.4$.

$$|\text{Error}| = |R_4(x)| \le \frac{\underset{[0,0.4]}{\text{Max}} |f^{(5)}(x)|\, x^5}{5!} \qquad |f^{(5)}(x)| = \sin x$$

$$= \frac{\sin(0.4)\ (0.4)^5}{5!} = 3.323036 \times 10^{-5}$$

Q_2. Use False position method to estimate the root of $\cos x - ln\, x + 1 = 0$ on the interval $[1, 2]$. Find 2 iterations.

a_n	c_n	b_n	$f(c_n)$
1^{+}	1.933747	2^{-}	−0.014494
1 +	1.92504	1.933747	

$$c_0 = b_0 - \frac{f(b_0)(b_0 - a_0)}{f(b_0) - f(a_0)} = 2 - \frac{f(2)(2-1)}{f(2) - f(1)} =$$

$$= 2 - \frac{(-0.10929)(2-1)}{(-0.10929) - 1.54030} \approx 1.933747$$

$$c_1 = 1.933747 - \frac{(-0.014494)(1.933747 - 1)}{(-0.014494 - 1.54030)} \approx 1.92504$$