جامعة بيرزيت
BIRZEIT UNIVERSITY
Mathematics Department

13/15

Math 330

Quiz 2
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Consider the fixed point iteration $p_{n+1} = \sqrt[4]{p_n + 1} = g(p_n)$.

a- Show that $g(x)$ has a fixed point in I = [1,2].
b- Show that if $p_0 \in I$, then the fixed point iteration converges.
c- Estimate the fixed point p starting with $p_0 = 1.5$, (do only 4 iterations)

a. $g(x) = \sqrt[4]{x+1} \Rightarrow f(x) = g(x) - x = \sqrt[4]{x+1} - x$ 1

Bolzano ✓

$f(x)$ is continuous on $[1,2]$ 1

$f(1) = \sqrt[4]{2} - 1 = 1.189207115 > 0$

$f(2) = \sqrt[4]{3} - 2 = -0.683926 < 0$ 1

inc −2

$\Rightarrow$ there's a root for $f(x) = \sqrt[4]{x+1} - x$ on $[1,2]$

$g(x) = \sqrt[4]{x+1} = x \Rightarrow \therefore g(x)$ has a fixed point in $[1,2]$

b. $K \leq \max |g'(x)| \leq 1$

$g'(x) = (x+1)^{-\frac{3}{4}} \left(\frac{1}{4}\right) = \frac{1}{4\sqrt[4]{(x+1)^3}}$ 1   decreasing 2

$\max |g'(x)| \Rightarrow g'(1) = \frac{1}{4\sqrt[4]{8}} = 0.148651 < 1$ 2

$\therefore \exists\, P_0 \in I$ while the fixed point iteration converges.

c. $P_0 = 1.5$

$P_1 = 1.257743349$

$P_2 = 1.225755182$

$P_3 = 1.221432153$

$P_4 = 1.220838632 \Rightarrow P$

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