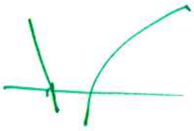
Math 330 Quiz 2

Consider the fixed point iteration $p_{n+1} = \ln(2p_n + 1) = g(p_n)$.

a- Show that $g(x)$ has a fixed point in I = [1 , 2]

$g(x) = \ln(2x+1)$ is increasing function 

4 $\quad g(1) = \ln 3 = 1.0986 \in [1,2]$

$g(2) = \ln 5 = 1.609 \in [1,2]$

since $g(x)$ is increasing so $g(1) \le g(x) \le g(2)$

3 $\quad \Rightarrow 1.0986 \le g(x) \le 1.609$

$\Rightarrow g(x) \in [1,2]$, so by theorem $g(x)$ has a fixed point in $[1,2]$

b- Show that if $p_0 \in I$, then the fixed point iteration converges.

2 $\quad |g'(x)| = \left|\frac{2}{2x+1}\right| = \frac{2}{2x+1}$ in $[1,2]$

for $1 \le x \le 2$

$3 \le 2x+1 \le 5$

3 $\quad \frac{1}{5} \le \frac{1}{2x+1} \le \frac{1}{3}$

$\frac{2}{5} \le \frac{2}{2x+1} \le \frac{2}{3}$

$\Rightarrow |g'(x)| = \frac{2}{2x+1} \le \frac{2}{3} < 1$

2 so by theorem the iteration converges for any $p_0 \in [1,2]$

c- Estimate the fixed point p starting with $p_0 = 1.2$, (do only 3 iterations)

3 $\quad p_1 = 1.224$

$p_2 = 1.238$

$p_3 = 1.246$

d- Find n so that the accuracy will be less than 10^{-5}.

$k = \frac{2}{3}$, $p_0 = 1.2$, $p_1 = 1.224$

3 $\quad |\text{Error}| \le \frac{k^n |p_1 - p_0|}{1-k} < 10^{-5}$

$\frac{\left(\frac{2}{3}\right)^n \cdot (0.024)}{\frac{1}{3}} < 10^{-5}$

$n \geq 21.9$

$n = 22$