Birzeit University-Dept. of Mathematics
Maec330-Microeconomic Theory

Second Hour Exam (Instructor: Dr. Marwan Aloqeili) Fall 2012/2013

Name: KEY Number:

10 **Question 1** Consider the problem of maximizing the function $f(x, y) = x^2 + 4x + 4y^2$ subject to the constraint $2x + 2y = 1$.

5 (a) Use second order conditions to show that the point $(x, y, \mu) = (0, 0.5, 2)$ is a minimum.

$$L(x,y,\mu) = x^2 + x + 4y^2 - \mu(2x + 2y - 1)$$

$$H = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 8 \end{pmatrix}$$

$$|H| = -32 - 8 = -40 < 0 \Rightarrow \text{min.}$$

5 (b) Use the envelope theorem to estimate the maximum of the same function under the constraint $2x+2y = 1.1$ and the maximum under the constraint $2x+2.1y = 1$.

(i) $2x + 2y = 1.1$

$$f^*_{new} = f(0, \tfrac{1}{2}) + \mu\,(0.1)$$

$$= 1 + 2(0.1) = 1.2.$$

(ii) $L = x^2 + 4x + 4y^2 - \mu(2x + ay - 1)$

$$f^*_{new} = f(0, \tfrac{1}{2}) + \frac{\partial L}{\partial a}\, da$$

$$= 1 - \mu y\,(0.1)$$

1

$$= 1 - (2)(\tfrac{1}{2})(0.1)$$

$$= 1 - 0.1 = 0.9.$$

Question 2 Answer the following:

(a) Use Euler's formula to show that the function $x^{1/3}y^{1/6}$ is homogeneous.

$$X\left(\frac{1}{3}X^{-2/3}y^{1/6}\right)+y\left(\frac{1}{6}X^{1/3}y^{-5/6}\right)$$

$$=\frac{1}{3}X^{1/3}y^{1/6}+\frac{1}{6}X^{1/3}y^{1/6}$$

$$=\frac{1}{2}X^{1/3}y^{1/6}$$

$\Rightarrow$ it is homog- of degree $1/2$

(b) Use f_x/f_y to show that the function $f(x,y)=\frac{x^2y^2}{xy+1}$ is homothetic.

$$f_X=\frac{(Xy+1)(2Xy^2)-y(X^2y^2)}{(Xy+1)^2}=\frac{X^2y^3+2Xy^2}{(Xy+1)^2}$$

$$f_y=\frac{(Xy+1)(2X^2y)-X(X^2y^2)}{(Xy+1)^2}=\frac{X^3y^2+2X^2y}{(Xy+1)^2}$$

$$\frac{f_X}{f_y}=\frac{X^2y^3+2Xy^2}{X^3y^2+2X^2y}=\frac{\cancel{Xy}(Xy^2+2y)}{\cancel{Xy}(X^2y+2X)}=\frac{y}{X}\;\frac{Xy+2}{Xy+2}$$

$$\frac{f_X(X,y)}{f_X(X,y)}=\frac{y}{X}$$ homogeneous of degree zero

$\Rightarrow f$ is homothetic