48/50 Excellent.

BIRZEIT UNIVERSITY

MATHEMATICS DEPARTMENT
MATH330 -Second Exam-
First semester 2019/2020

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(Q1) [5 points] Use the nodes $x_0 = 0,\ x_1 = 2,\ x_2 = 3,\ x_3 = 4$ to find Lagrange coefficient $L_{3,3}$ at $x = 1$

$$L_{3,3} = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$\frac{(1-0)(1-2)(1-3)}{(4-0)(4-2)(4-3)}$$

$$\frac{2}{4\times 2\times 1} = \frac{2}{8} = .25$$

5

(Q2) [5 points] Let $f(x) = \ln(x+1)$, $3 \le x \le 3.5$. Assuming a uniform partition, find the upper bound for the interpolating error $E_2(x)$

$$h = \frac{3.5-3}{2} = .25$$

$$E_2(x) \le \frac{h^3\ M_3}{9\sqrt{3}}$$

$$= \frac{(.25)^3 \times .03125}{9\times\sqrt{3}}$$

5

$$E_2(x) = \frac{.015625 \times .03125}{9\cdot\sqrt{3}}$$

$$|E_2(x)| \le 3.1323 \times 10^{-5}$$

$$f'(x) = \frac{1}{x+1}$$

$$f''(x) = \frac{-1}{(x+1)^2}$$

$$f'''(x) = \frac{2 \cancel{(x+1)}}{(x+1)^3}$$

$f'''(x)$ is decreasing from know the behavior

Max at $x = 3$

$$M_3 = \frac{2}{(3+1)^3} = \frac{2}{64} = .03125$$

(Q3) [5 points] Let $f(x) = \cos(\pi x)$, $0 \le x \le 2$. With equally spaced nodes, estimate f(1.5) using Newton's polynomial $P_2(x)$

$x = \frac{2-0}{2} = 1$

x_k	$f(x_k)$	$f[x_{k-1}, x_k]$	$f[x_{k-2}, x_{k-1}, x_k]$
0	1 a_0	—	—
1	-1	-2 a_1	—
2	1	2	2 a_2

$P_2(x) = a_0 + a_1(x - x_0) + a_2(x-x_0)(x-x_1)$

$= 1 + -2(1.5-0) + 2(1.5-0)(1.5-1)$

$= 1 - 3 + 1.5 = -.5$

5

(Q4) [5 points] Use Gauss-Seidel iteration with $(p_0, q_0) = (3, 2)$ to find (p_1, q_1) for the following system

$$g_1(x,y) = \sqrt{x+1} + \frac{2}{y+2}$$

$$g_2(x,y) = \frac{x}{10} + 2y$$

$p_1 = g_1(p_0, q_0) = \sqrt{3+1} + \frac{2}{2+2} = 2 + .5 = 2.5$

$q_1 = g_1(p_1, q_0) = \frac{2.5}{10} + 4 = 4.25$

$(p_1, q_1) = (2.5, 4.25)$

5

(Q5) [5 points] Let A be a 3×3 matrix. Find the cost of evaluating $2A^3$

~~total Cost~~ cost $= n^2 +$

Cost for find $A^3 = (3-1)(2n^3 - n^2) = 2(2 \times 3^3 - 3^2)$

$= 90$

Cost for find $2A^3 = n^2 = 3^2 = 9$

total cost for finding $2A^3 = 9 + 90 = 99$

5

(Q6) [5 points] Prove the uniform interpolating error bound: $E_1(x) \leq \frac{h^2 M_2}{8}$, where $h = x_1 - x_0$ and $M_2 = \max |f''(x)|$ on $[x_0, x_1]$

$$E_n(x) = \left| \frac{(x-x_0) \cdots (x-x_n) \, f^{n+1}(c)}{(n+1)!} \right|$$

as $n=1$

$$E_1(x) = \left| \frac{(x-x_0)(x-x_1) \, f^{(2)}(c)}{2!} \right|, \text{ where } c \max [x_0, x_1]$$

let $g(x) = (x-x_0)(x-x_1)$

$g'(x) = (x-x_0) + x - x_1$

$g'(x) = 0$

$0 = x - x_0 + x - x_1$

$2x = x_1 + x_0$

$x = \frac{x_1 + x_0}{2}$ — ①

$f^2(c) = \max |f''(x)|$ on $[x_0, x_1]$

$= M_2$

$$E_1(x) = \frac{\left(\frac{x_1 + x_0 - x_0}{2}\right)\left(\frac{x_1 + x_0 - x_1}{2}\right) f^{(2)}(c)}{2!}$$

$$E_1(x) = \frac{\left(\frac{x_1}{2} - \frac{x_0}{2}\right)\left(\frac{x_1}{2} + \frac{x_0}{2}\right) f^{(2)}(c)}{}$$

5

$$E_1 = \left| \frac{(x_1 - x_0)}{2} \cdot \frac{(x_1 - x_0)}{2} M_2 \right|$$

$$E_1 = \left| \frac{(x_1 - x_0)(x_1 - x_0) M_2}{4 \times 2!} \right|$$

$h \quad 2 \; !$

$E_1(x)$ ال تعوضنا عن ① معادلة

نتبع

$E_1 = \frac{h^2 M_2}{8}$

(Q7) [10 points] Use Newton's method with $(p_0, q_0) = (0, 1)$ to find (p_1, q_1) for the following system.

$$x^2y + \cos x + 2e^x = 0$$

$$x\ln y + \frac{1}{y} + 1 = 0$$

$$J = \begin{vmatrix} \frac{df_1}{dx} & \frac{df_1}{dy} \\ \frac{df_2}{dx} & \frac{df_2}{dy} \end{vmatrix}$$

$$J = \begin{vmatrix} 2yx + \sin x + 2e^x & x^2 \\ \ln y & \frac{x}{y} - \frac{1}{y^2} \end{vmatrix}$$

$$J_{(0,1)} \begin{vmatrix} \overset{2}{\cancel{1.84147 + 2}} & 0 \\ 0 & -1 \end{vmatrix} = \begin{vmatrix} \overset{2}{\cancel{1.158529}} & 0 \\ 0 & -1 \end{vmatrix}$$

$$\det J^{-1} = \overset{-2}{\cancel{-1.158529}} - 0 = \overset{-2}{\cancel{-1.158529}}$$

$$J^{-1} = \frac{-1}{\underset{2}{\cancel{1.158529}}} \begin{bmatrix} \overset{-1}{\cancel{1.158529}} & 0 \\ 0 & \underset{2}{\cancel{1.158529}} \end{bmatrix}$$

$F_1(p_0, q_0) =$

$0 + \cos 0 + 2e^0$

$= 3$

$F_2(p_0, q_0) = 0$

$+1 + 1 = 2$

$$\begin{bmatrix} p_1 \\ q_1 \end{bmatrix} = \begin{bmatrix} p_0 \\ q_0 \end{bmatrix} - J^{-1} \begin{bmatrix} F_1(p_0, q_0) \\ F_2(p_0, q_0) \end{bmatrix}$$

10

$$\begin{bmatrix} p_1 \\ q_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} \overset{\frac{1}{2}}{\cancel{.863163}} & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} p_1 \\ q_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} \overset{3/2}{\cancel{2.5894996679}} \\ -2 \end{bmatrix} = \begin{bmatrix} \overset{-1.5}{\cancel{-2.5894996679}} \\ 3 \end{bmatrix}$$

$$(p_1, q_1) = (\overset{-1.5}{\cancel{-2.5894996679}}, 3) \quad (-1.5, 3)$$

(Q8) [10 points]

(a) Use Gaussian elimination with pivoting and two-digit rounding to solve the system below.

$$\begin{aligned} 2x_1 &+ x_2 + x_3 = 3 \\ 2x_1 &+ 2x_2 - x_3 = -2 \\ 8x_1 &- x_2 + x_3 = 11 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 8 & -1 & 1 & 11 \\ 2 & 2 & -1 & -2 \\ 2 & 1 & 1 & 3 \end{array}\right] = \left[\begin{array}{ccc|c} 8 & -1 & 1 & 11 \\ 0 & 2.25 & -1.25 & -4.75 \\ 0 & 1.25 & .75 & .25 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 8 & -1 & 1 & 11 \\ 0 & 2.25 & -1.25 & -4.75 \\ 0 & 0 & 1.44 & 2.89 \end{array}\right]$$

5

$x_3 = 2.0001 = 2$

$x_2 = \dfrac{-4.75 + 1.25 x_3}{2.25} = \dfrac{-2.25}{2.25} = -1$

$x_1 = \dfrac{11 - x_3 + x_2}{8} = 1$

$x_1 = 1, \ x_2 = -1, \ x_3 = 2$

(b) Find the cost of Gaussian elimination for a 3×3 system.

Total cost Gaussian elimination $= \frac{2}{3} n^3 + \frac{3}{2} n^2 - \frac{7}{6} n$

$= \frac{2}{3} \times 27 + \frac{3}{2} \times 9 - \frac{7}{6} \times 3$

$= 18 + 13.5 - 3.5$

$= 28$

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