BIRZEIT UNIVERSITY

Key

Mathematics Department
Math 330
Test 2

1st Semester 21-22

Student name: .... ID no.: ...... sec........

Note (For all problems, write three digits arthmetics rounding)

Q#1)(6 points) Consider the data

$(1, f(1)), (1.2, f(1.2)), (1.3, f(1.3))$ Where $f(x) = \sqrt{x+2}$

Find Lagrange interpolation polynomial $p_2(x)$ and use it to estimate $f(1.25)$

1 The data $(1, 1.73), (1.2, 1.79), (1.3, 1.82)$

1 $P_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$

1 $= \frac{(x-1.2)(x-1.3)}{(-0.2)(-0.3)}(1.73) + \frac{(x-1)(x-1.3)}{(0.2)(-0.1)}(1.79) + \frac{(x-1)(x-1.2)}{(0.3)(0.1)}(1.82)$

2 $= 28.8(x-1.2)(x-1.3) - 89.5(x-1)(x-1.3) + 60.7(x-1)(x-1.2)$

1 $f(1.25) \approx P_2(1.25) = -0.072 + 1.12 + 0.759$

≈ 1.81

Q#2)(12 points)

(a) Consider the data

$(1, f(1)), (1.4, f(1.4)), (1.8, f(1.8)), (2.2, f(2.2))$ Where $f(x) = \sqrt{x+2}$

Find Newton interpolation polynomial $p_3(x)$ and use it to estimate $f(1.25)$

(b) Find the best upper bound for $E_3(x)$ above.

me data: (1, 1.73), (1.4, 1.84), (1.8, 1.95), (2.2, 2.05)

x_k	y_k	1st	2nd	3rd
1	1.73			
1.4	1.84	0.275		
1.8	1.95	0.275	0	
2.2	2.05	0.25	-0.313	-0.261

$a_0 = 1.73$

$a_1 = 0.275$ (2)

$a_2 = 0$

$a_3 = -0.261$

$p_3(x) = 1.73 + 0.275(x-1) - 0.261(x-1)(x-1.4)(x-1.8)$

$f(1.25) \approx p_3(1.25) \approx 1.80 \quad (1.795520)$

$|E_3(x)| \le \frac{h^4 M_4}{24}$

$h = 0.4$ (1)

$M_4 = 0.02$

$|E_3(x)| \le \frac{(0.4)^4 (0.02)}{24}$ (1)

$= 0.0000213$

$f = (x+2)^{1/2}$

$f' = \frac{1}{2}(x+2)^{-1/2}$

$f'' = -\frac{1}{4}(x+2)^{-3/2}$

$f''' = \frac{3}{8}(x+2)^{-5/2}$

$f^{(4)}(x) = \frac{15}{16}(x+2)^{-7/2}$

$= \frac{15}{16} \frac{1}{(x+2)^{7/2}}$ (2)

$M_4 = \frac{15}{16} \frac{1}{3^{7/2}} \approx 0.02$

Q#3)(12 points) Consider the data

$$(1,-2),(2,2.5),(3,8)$$

(a) Derive the normal equations for the best fit of the form $f(x)=\frac{C}{x}+Dx^2$

(b) Find C, D using the normal equations derived above and the given data.

$$E(C,D)=\sum_{k=1}^{n}\left(\frac{C}{x_k}+Dx_k^2-y_k\right)^2$$

(2) $$\frac{\partial E}{\partial C}=0=\sum_{k=1}^{n}2\left(\frac{C}{x_k}+Dx_k^2-y_k\right)\cdot\frac{1}{x_k}$$

$$\Rightarrow \boxed{C\sum_{k=1}^{n}\frac{1}{x_k^2}+D\sum_{k=1}^{n}x_k=\sum\frac{y_k}{x_k}} \quad ---(1)$$

(2) $$\frac{\partial E}{\partial D}=0=\sum 2\left(\frac{C}{x_k}+Dx_k^2-y_k\right)x_k^2$$

$$\boxed{C\sum_{k=1}^{n}x_k+D\sum_{k=1}^{n}x_k^4=\sum_{k=1}^{n}y_k x_k^2} \quad ---(2)$$

	x_k	y_k	$\frac{1}{x_k^2}$	$\frac{y_k}{x_k}$	x_k^4	$y_k x_k^2$
	1	-2	1	-2	1	-2
	2	2.5	0.25	1.25	16	10
	3	8	0.111	2.67	81	72
Total	6		1.36	1.92	98	80

(4)

(2) $$\begin{cases}1.36\,C+6D=1.92\\ 6C+98D=80\end{cases}$$

$C \simeq -3.00$ X

(2) $D \simeq 1$

$$y=\frac{3.00}{x}+x^2$$

Q#4) (12 points) Find the curve fit $f(x) = \frac{x}{Cx+D}$ for the following data (1,0.250), (2,0.286),(3,0.300) using a suitable **Linearization**

$$y = \frac{x}{Cx+D}$$

$$\frac{1}{y} = \frac{Cx+D}{x} = C + \frac{D}{x} = D\left(\frac{1}{x}\right) + C$$

4

$$Y \qquad = AX + B$$

$$\Rightarrow Y = \frac{1}{y}, \quad X = \frac{1}{x} \Rightarrow A, B$$

then $D = A$, $C = B$

4

x	y	$X = \frac{1}{x}$	$Y = \frac{1}{y}$	XY	X^2
1	0.25	1	4	4	1
2	0.286	0.5	3.5	1.75	0.25
3	0.300	0.333	3.33	1.11	0.111
		1.83	10.8	6.86	1.36

$$A \sum_{k=1}^{n} X_k^2 + B \sum_{k=1}^{n} X_k = \sum_{k=1}^{n} X_k Y_k$$

$$A \sum_{k=1}^{n} X_k + 3B = \sum_{k=1}^{n} Y_k$$

4

$$1.36A + 1.83B = 6.86$$

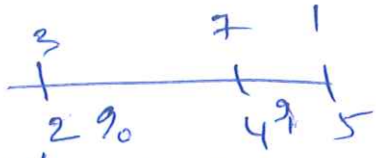
$$1.83A + 3B = 10.8$$

$$A \simeq 1.12 \qquad \Rightarrow \qquad D = A = 1.12$$

$$B \simeq 2.91 \qquad\qquad C = B = 2.91 \Rightarrow y \simeq \frac{x}{2.91x + 1.12}$$

Q#5)(8 points) Find the clamped spline for the following data
$(2,3), (4,7), (5,1), \quad f'(2) = 2, f'(5) = 10$

(Just set up the equations)



$$g(x) = \begin{cases} g_0(x) = a_0(x-2)^3 + b_0(x-2)^2 + c_0(x-2) + d_0 & \text{on } [2,4] \\ g_1(x) = a_1(x-4)^3 + b_1(x-4)^2 + c_1(x-4) + d_1 & \text{on } [4,5] \end{cases}$$

$$g'(x) = \begin{cases} g_0'(x) = 3a_0(x-2)^2 + 2b_0(x-2) + c_0 & \text{on } (2,4) \\ g_1'(x) = 3a_1(x-4)^2 + 2b_1(x-4) + c_1 & \text{on } (4,5) \end{cases}$$

$$g''(x) = \begin{cases} g_0''(x) = 6a_0(x-2) + 2b_0 & \text{on } (2,4) \\ g_1''(x) = 6a_1(x-4) + 2b_1 & \text{on } (4,5) \end{cases}$$

$g_0(2) = 3 \Rightarrow d_0 = 3$ --- (1)

$g_1(4) = 7 \Rightarrow d_1 = 7$ --- (2)

$g_1(5) = 1 \Rightarrow a_1 + b_1 + c_1 + d_1 = 1$ --- (3)

$g_0(4) = g_1(4)$

$8a_0 + 4b_0 + 2c_0 + d_0 = d_1$ --- (4)

$g_0'(4) = g_1'(4)$

$12a_0 + 4b_0 + c_0 = c_1$ --- (5)

$g_0''(4) = g_1''(4)$

$12a_0 + 2b_0 = 2b_1$ --- (6)

$g_0'(2) = 2 \Rightarrow c_0 = 2$ --- (7)

$g_1'(5) = 10 \Rightarrow 3a_1 + 2b_1 + c_1 = 10$ --- (8)

(1 point for each equation)