جامعة بيرزيت
BIRZEIT UNIVERSITY

Department of Mathematics

Homework

Differential Equations (Math 331) **First Semester 2019/2020**

Instructor: Dr. Ala Talahmeh **Section 2.3**

Q1 [Problem 12 of Section 1.2 Page 17].

A radioactive material, such as the isotope thorium-234, disintegrates at a rate proportional to the amount currently present. If $Q(t)$ is the amount present at time t, then

$$\frac{dQ}{dt} = -rQ,$$

where $r > 0$ is the decay rate.
(a) If 100 mg of thorium-234 decays to 82.04 mg in 1 week, determine the decay rate r.
(b) Find an expression for the amount of thorium-234 present at any time t.
(c) Find the time required for the thorium-234 to decay to one-half its original amount.

Q2 [Problem 11 of Section 1.2 Page 17].

Consider the falling object of mass $10kg$ in Example 2, but assume now that the drag force is proportional to the square of the velocity.
(a) If the limiting velocity is $49m/s$ (the same as in Example 2), show that the equation of motion can be written as

$$\frac{dv}{dt} = [(49)^2 - v^2]/245.$$

(b) If $v(0) = 0$, find an expression for $v(t)$ at any time.
(e) Find the distance $x(t)$ that the object falls in time t.
(f) Find the time T it takes the object to fall $300m$.

Q3 [Problem 23 of Section 1.1 Page 9].

Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between the temperature of the object itself and the temperature of its surroundings (the ambient air temperature in most cases). Suppose that the ambient temperature is $70°F$ and that the rate constant is $0.05(\text{min})^{-1}$. Write a differential equation for the temperature of the object at any time. Note that the differential equation is the same whether the temperature of the object is above or below the ambient temperature.

Q4 [Problem 15 of Section 1.2 Page 17].

According to Newton's law of cooling (see Problem 23 of Section 1.1), the temperature $u(t)$ of an object satisfies the differential equation

$$\frac{du}{dt} = -k(u - T),$$

where T is the constant ambient temperature and k is a positive constant. Suppose that the initial temperature of the object is $u(0) = u_0$.
(a) Find the temperature of the object at any time.
(b) Let τ be the time at which the initial temperature difference $u_0 - T$ has been reduced by half. Find the relation between k and τ .

Q5 [Problem 16 of Section 2.3 Page 62].

Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between its temperature and that of its surroundings. Suppose that the temperature of a cup of coffee obeys Newton's law of cooling. If the coffee has a temperature of $200^\circ F$ when freshly poured, and 1 min later has cooled to $190^\circ F$ in a room at $70^\circ F$, determine when the coffee reaches a temperature of $150^\circ F$.

Q6 [Problem 20 of Section 2.3 Page 64].

A ball with mass 0.15 kg is thrown upward with initial velocity 20 m/s from the roof of a building 30 m high. Neglect air resistance.
(a) Find the maximum height above the ground that the ball reaches.
(b) Assuming that the ball misses the building on the way down, find the time that it hits the ground.

Q7 [Example 1 of Section 2.3 Page 52].

At time $t = 0$ a tank contains Q_0 lb of salt dissolved in 100 gal of water. Assume that water containing $\frac{1}{4}$lb of salt/gal is entering the tank at a rate of r gal/min and that the well-stirred mixture is draining from the tank at the same rate.
(a) Set up the initial value problem that describes this flow process.
(b) Find the amount of salt $Q(t)$ in the tank at any time.
(c) Find the limiting amount Q_L that is present after a very long time.
(d) If $r = 3$ and $Q_0 = 2Q_L$, find the time T after which the salt level is within 2% of Q_L.

Q8 [Example 3 of Section 2.3 Page 57].

Consider a pond that initially contains 10 million gal of fresh water. Water containing an undesirable chemical flows into the pond at the rate of 5 million gal/yr, and the mixture in the pond flows out at the same rate. The concentration $\gamma(t)$ of chemical in the incoming water varies periodically with time according to the expression $\gamma(t) = 2 + \sin 2t$ g/gal. Construct a mathematical model of this flow process and determine the amount of chemical in the pond at any time.

Q9 [Summer 2018].

Let P be the fish population in a certain large lake. Assume that the rate of increase in the population due to births is 40 % per year and the death rate is 10% per year. Fishermen harvest the fish at the rate of $10,000$ fish per year. Construct a mathematical model of this model and determine the fish population at any time.

Good Luck