Key

Birzeit University
Mathematics Department
Math331
Quiz 5

Instructor: Dr. Ala Talahmeh
Name:..
Section: (5)

First Semester 2019/2020
Number:..............
Date: 12/11/2019

Question I [6 points]. If $y_1 = x\cos x$ is one solution of

$$x^2y'' - 2xy' + (x^2+2)y = 0, \quad x > 0.$$

Find the second solution of this differential equation.

Question II [4 points]. Solve the DE

$$x^2y'' - 3xy' + 4y = 0, \quad x > 0.$$

Sol. Question I. $y'' - \frac{2}{x}y' + \left(\frac{x^2+2}{x^2}\right)y = 0$

$\boxed{p(x) = -\frac{2}{x}}$ $\quad y_2 = y_1 \int \frac{W}{y_1^2}\,dx$

$\cdot\, W(y_1, y_2) = c e^{-\int -\frac{2}{x}dx} = cx^2, \; x > 0.$

$\therefore y_2 = x\cos x \int \frac{x^2}{x^2\cos^2 x}\,dx = x\cos x \tan x = x\sin x.$

QII. Cauchy-Euler Eq.

the aux. eq. is $m^2 + (-3-1)m + 4 = 0$

$\Rightarrow (m-2)^2 = 0 \Rightarrow m = 2, 2$

$\therefore y = c_1x^2 + c_2x^2\ln x, \quad x > 0.$

Good Luck

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Name:..
Section: (1)

First Semester 2019/2020
Number:..............
Date: 11/11/2019

Question I [5 points]. Let $y(t)$ be the solution of the IVP

$$y'' + y' - 2y = 0, \;\; y(0) = \alpha, \;\; y'(0) = 2.$$

Find the value of α for which $\lim_{t\to\infty} y(t) = 0$.

Question II [5 points]. Verify that $y_1 = t$, and $y_2 = te^t$ form a fundamental set of solutions for the DE

$$t^2y'' - t(t+2)y' + (t+2)y = 0, \;\; t > 0.$$

Sol. Question I. The aux. eq. is $r^2+r-2=0$

$(r+2)(r-1)=0 \Rightarrow r=-2, 1$

$\boxed{y = c_1 e^{-2t} + c_2 e^t}$

$y(0) = \boxed{c_1 + c_2 = \alpha}$ --- ①, $y' = -2c_1 e^{-2t} + c_2 e^t$

$y'(0) = \boxed{-2c_1 + c_2 = 2}$ — ②

From ① & ②, $\boxed{c_1 = \frac{\alpha-2}{3}}$ $\boxed{c_2 = \frac{2\alpha+2}{3}}$

$\therefore y = \frac{\alpha-2}{3} e^{-2t} + \frac{2\alpha+2}{3} e^t$

Since $\lim_{t\to\infty} y = 0 \Rightarrow \frac{2\alpha+2}{3} = 0 \Rightarrow \boxed{\alpha = -1}$

QII. For $y_1 = t$, $y_1' = 1$, $y_1'' = 0$

$L.H.S = 0 - t(t+2) + (t+2)t = 0 \Rightarrow y_1 = t$ is a sol.

For $y_2 = te^t$

$y_2' = (t+1)e^t$

$y_2'' = (t+2)e^t$

Good Luck

$$\text{L.H.S} = t^2(t+2)e^t - t(t+2)(t+1)e^t + (t+2)te^t$$

$$= t^2\cancel{(t+2)}e^t - t^2\cancel{(t+2)}e^t - t\cancel{(t+2)}e^t + \cancel{t(t+2)}e^t$$

$$= 0 \Rightarrow y_2 = te^t \text{ is a sol.}$$

- $W(y_1, y_2) = \begin{vmatrix} t & te^t \\ 1 & (t+1)e^t \end{vmatrix}$

$$= t(t+1)e^t - te^t$$

$$= t^2e^t \neq 0, \text{ since } t>0.$$

$\therefore \{y_1, y_2\}$ are lin. indep.

Hence $\{y_1, y_2\}$ form a fundamental set of solutions for the given D.E.