Key

Birzeit University
Mathematics Department
Math331
Quiz 2

Instructor: Dr. Ala Talahmeh
Name:...
Section: (5)

First Semester 2019/2020
Number:...............
Date: 26/09/2019

Question I [10 points]. Find an **explicit solution** of the following differential equation

$$x\frac{dy}{dx} + 2y + x^3y^2\cos x = 0.$$

$\frac{dy}{dx} + \frac{2}{x}y = -x^2\cos x \cdot y^2$ 1

Bernoulli's DE with $n=2$ 1

Divide by y^2: $\boxed{\frac{1}{y^2}\frac{dy}{dx} + \frac{2}{x}y^{-1} = -x^2\cos x}$ ($*$)

Let $u = y^{1-n} \Rightarrow u = y^{-1}$ 1 $\Rightarrow \frac{du}{dx} = -y^{-2}\frac{dy}{dx}$ 1

Substitute in ($*$):

$\boxed{-\frac{du}{dx} + \frac{2}{x}u = -x^2\cos x}$ 1

$\Rightarrow \boxed{\frac{du}{dx} - \frac{2}{x}u = x^2\cos x}$ 1 lin. DE in u

$\mu(x) = e^{-\int \frac{2}{x}dx} = \frac{1}{x^2}, \quad x>0$ 1

2 $u = x^2\left[\int \cancel{x^2}\cos x \cdot \frac{1}{\cancel{x^2}}dx + c\right] = x^2\sin x + x^2 c$

Good Luck

$y^{-1} = x^2\sin x + cx^2 \Rightarrow y = \frac{1}{x^2(\sin x + c)}$ 1 key

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Quiz 1

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Question I [10 points].

a. If $\lim_{t\to\infty} y(t) = 2$, for any solution of the differential equation $y' = \alpha y + 2$, then find the value(s) of α.

4

$y' = 0 \Rightarrow \alpha y + 2 = 0 \Rightarrow -\frac{2}{\alpha} = 2$

$\therefore \alpha = -1$

b. Verify that the function

$$y = 3e^{-x} + e^{x} + \frac{1}{2}x\sinh x$$

is a solution of the second order differential equation

$$y'' - y = \cosh x.$$

2 $y' = -3e^{-x} + e^{x} + \frac{1}{2}\sinh x + \frac{1}{2}x\cosh x$

2 $y'' = 3e^{-x} + e^{x} + \frac{1}{2}\cosh x + \frac{1}{2}\cosh x + \frac{1}{2}x\sinh x$

$\therefore y'' - y = \cancel{3e^{-x}} + \cancel{e^{x}} + \cosh x + \cancel{\frac{1}{2}x\sinh x}$

2 $\cancel{-3e^{-x}} - \cancel{e^{x}} - \cancel{\frac{1}{2}x\sinh x}$

$= \cosh x$

Good Luck