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Birzeit University
Mathematics Department
Math331
Quiz 3

Instructor: Dr. Ala Talahmeh | **First Semester 2019/2020**
Name:..... | **Number:**.....
Section: (1) | **Date: 07/10/2019**

Question I [10 points]. A radioactive material, such as the isotope thorium-234, disintegrates at a rate proportional to the amount currently present. If $Q(t)$ is the amount present at time t, then

$$\frac{dQ}{dt} = -rQ,$$

where $r > 0$ is the decay rate.

a. If 100 mg of thorium-234 decays to 82.04 mg in 1 week, determine the decay rate r.

b. Find an expression for the amount of thorium-234 present at any time t.

Sol. we need to solve $\begin{cases} \frac{d\varphi}{dt} = -r\varphi \\ \varphi(0) = 100, \ \varphi(7) = 82.04 \end{cases}$ (2)

Indeed, $\int \frac{d\varphi}{\varphi} = \int -r\,dt \Rightarrow \ln|\varphi(t)| = -rt + c$ (2)

$\Rightarrow \boxed{\varphi(t) = Ae^{-rt}}$ (2)

$\varphi(0) = \boxed{A = 100}$ (1)

$\varphi(7) = 100\,e^{-7r} = 82.04 \Rightarrow \boxed{r = -\frac{1}{7}\ln\left(\frac{82.04}{100}\right)}$ (2) part (a)

$\therefore \varphi(t) = 100\,e^{\frac{1}{7}\ln(0.8204)t}$ (1)

or $\boxed{\varphi(t) = 100\,(0.8204)^{\frac{t}{7}}}$. part (b)

Good Luck

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Name:..... | **Number:**.....
Section: (5) | **Date: 08/10/2019**

Question I [10 points]. Consider a tank used in certain hydrodynamic experiments. After one experiment the tank contains 150 L of a dye solution with a concentration of 1 g/L. To prepare for the next experiment, the tank is to be rinsed with fresh water flowing in at a rate of 2 L/min, the well-stirred solution flowing out at the same rate. Find the time that will elapse before the concentration of dye in the tank reaches 1% of its original value.

Sol. Let $Q(t)$ be the amount of dye for all t.

$$\frac{dQ}{dt} = \text{rate in} - \text{rate out}$$

$$= 0 - \left(\frac{Q}{150}\right)(2)$$

$$\Rightarrow \boxed{\frac{dQ}{dt} = -\frac{1}{75}Q, \quad Q(0) = 150} \quad (4)$$

or $$\int \frac{dQ}{Q} = -\int \frac{1}{75}\,dt \Rightarrow \ln|Q(t)| = -\frac{1}{75}t + C$$

$$\Rightarrow \boxed{Q(t) = Ae^{-\frac{1}{75}t}} \quad (2)$$

$$Q(0) = \boxed{A = 150} \quad (1)$$

$$\therefore \boxed{Q(t) = 150e^{-\frac{1}{75}t}} \quad (1)$$

Now we need to find t such that $Q(t) = \frac{1}{100} \times 150 = 1.5$

$$\therefore 1.5 = 150e^{-\frac{1}{75}t} \Rightarrow -\frac{1}{75}t = \ln(0.01) \quad (2)$$

Good Luck

$$\Rightarrow \boxed{t = 75\ln(100)}$$