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Test # 2 Math 331 First Semester 2012/2013

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Question 1

Find the general solution for equation:

$$y''+y=\sec t$$

$r^2+1=0 \quad r^2=\mp i \quad y_c = C_1 \sin x + C_2 \cos x$

$W \begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix}$

$-\sin^2 x - \cos^2 x$

$= -1$

$y_{p_1} = -\sin x \int^t \frac{\cos x \frac{1}{\cos x}}{-+1} dx -$

$= +\sin x \int^t \frac{+\cos x}{\cos x} dx \Rightarrow +x \sin x$

$y_{p_2} = \cos x \int^t \frac{\sin x}{\cos x} dx \Rightarrow$

$-\cos x \ln|\cos x|$

$y_p = +x \sin x + -\cos x \ln|\cos x|$

$\boxed{y_p = x \sin x - \cos x \ln|\cos x|}$

$u_1 = \cos x$

$du_1 = -\sin x$

$u_2 = \cos x$

$du_2 = -\sin x$

$\cos^2 x + \sin^2 x = 1$

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Question 2

Determine a suitable form for a particular solution $Y(t)$ of the equation:

$$y^{(4)}+4y''=\sin 2t+te^t+4$$

Do not evaluate the constants.

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$r^4+4r^2=0 \quad r^2(r^2+4)=0 \quad r^2=0 \quad r^2=\mp 2i$

$y_c = C_1 + C_2 x + C_3 \sin 2x + C_4 \cos 2x$

$y_p = A t \sin 2t + B t \sin 2t + (Cx+d)e^t + E$

$r^2(r^2+4)$

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$r^2 + (\alpha - 1) r + \frac{5}{2} = 0 \Rightarrow r^2 + \alpha r - r + \frac{5}{2} = 0$

$r_1 r_2 = \frac{5}{2} \Rightarrow r_1 = \frac{5}{2 r_2}$

$r_1 + r_2 = \alpha - 1 \Rightarrow \frac{5}{2 r_2} + r_2 = \alpha - 1$

$\Rightarrow \frac{5}{2 r_2} + \frac{(r_2)^2}{2 r_2} = \alpha - 1$

$\Rightarrow \frac{5 + 2(r_2)^2}{2 r_2} = \alpha - 1$

$\Rightarrow 5 + 2 r_2^2 = 2 r_2 \alpha - 2 r_2$

$\alpha = \frac{5 + 2(r_2)^2}{2 r_2} - 1$

$r_1 r_2 = \frac{5}{2}$

$r_1 + r_2 = \alpha - 1$

$r_1 = \alpha - 1 - r_2$

$C_1 x^{r_1} + C_2 x^{r_2}$

$C_1 x^{r_1} + C_2 x^{\frac{5}{2 r_2}}$

$C_1 x^{\alpha - 1 - r_2} + C_2 x^{-r_2}$

$x^{\alpha - 1 - r^2} + x^{r_2} \Rightarrow$

$\lim_{r \to \infty}$

$r_1 + r_2 = \alpha - 1$

$r_2 = \alpha - 1 - r_1$

$r_1 + \alpha + 1 = -r^2$

$\begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix}$

$\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$

$\cos^2 x + \sin^2 x$

$= 1$

Question 3

$t^2 + (\alpha - 1) t + \frac{5}{2} = 0$

$r_1 r_2 = \frac{5}{2}$ $\quad r_1 = \frac{5}{2 r_2}$

$r_1 + r_2 = \alpha - 1$

$C_1 x^{\alpha - 1 - r_2} + C_2 x^{r_2} \Rightarrow$ r_2 must be less than 1 to be

$\lim_{r \to \infty} C_1 x^{\alpha - 1 + r_2} + C_2 x^{-r_2} \Rightarrow$ so

$\alpha - 1 + r_2 = 0$

$\lim_{r \to \infty}$

$\Rightarrow \alpha = 1 + r_2$ $\quad r_2$ less than 0

$r^2 + (\alpha - 1) + \frac{5}{2}$

Question 3

Consider the initial-value problem

$$t^2 y'' + \alpha t y' + (5/2) y = 0$$

$$\frac{-(\alpha-1) \mp \sqrt{(\alpha-1)^2 - 4 \cdot \frac{5}{2}}}{2} \qquad \frac{-(\alpha-1) \mp \sqrt{(\alpha-1)^2 - 10}}{2}$$

3.3 Find all values of α for which all solutions approach zero as $t \to \infty$

$y = C_1 x^{r_1} + C_2 x^{r_2}$

$\cancel{t\, r^2 + (\alpha - 1) r + \frac{5}{2} = 0}$ $\qquad t^2 + (\alpha - 1) t + \frac{5}{2} = 0$

$\cancel{(\alpha - 1) = \alpha\ ...}$ $\qquad \cancel{x_1 + x_2 = \alpha - 1}$, $\cancel{x_1 x_2 = \frac{5}{2}}$

$r^2 + (\alpha - 1) r + \frac{5}{2} = 0$

$\cancel{\alpha \leq 0}$

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$\cancel{x_1 = \frac{5}{2x_2}}$

$\cancel{\frac{5}{2x_2} + x_2 = \alpha - 1}$

$\cancel{\frac{5 + x_2^2}{2x_2} = \alpha - 1}$

$\cancel{2x_2(\alpha - 1) = 5 + x_2^2}$

$\cancel{\Rightarrow 2x_2\alpha - 2x_2 = 5 + x_2^2}$

$x \to \infty$

r_1, r_2 both neg

$\frac{-(\alpha-1)}{2} \mp \frac{\sqrt{(\alpha-1)^2 - 10}}{2}$

$-\frac{(\alpha-1)}{2} < 0$

$\alpha > 1$

Question 4

Find the general solution for equation:

$t = r$ $\qquad t^2 y'' - 4ty' + 6y = 0.$

$r^2 + (-4 - 1) r + 6 = 0 \Rightarrow r^2 - 5r + 6 = 0$

$(r - 2)(r - 3)$

$y = C_1 x^2 + C_2 x^3$

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Question 5

Find the largest interval on which the initial-value problem

$$(t^2 - 1) y'' + (\sin t) y' + (\cos t) y = 0, \quad y(5) = 0, \; y'(5) = 1$$

is certain to have a unique solution

$t^2 - 1 = 0 \qquad t^2 = 1 \qquad t = \mp 1$

$\cos t$, $\sin t$ Continious every where

$t \in (1, \infty)$

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Question 6

Find the first 3 nonzero terms in each of two power series solutions about the origin for the differential equation:

$$e^x y'' + xy = 0$$

$y(0) = \phi(0) = \boxed{a_0}$

$y'(0) = \phi'(0) = \boxed{a_1}$

$y''(0) = \phi''(0) = \frac{-x\,y}{e^x} = 0$

$y'''(0) = \phi'''(0) \Rightarrow e^x y'' + e^x y''' + x y' + y = 0$

$\Rightarrow y''' = \frac{-y - x y' - e^x y''}{e^x} = \frac{-a_0}{1} \Rightarrow \boxed{a_3 = \frac{-a_0}{3!}}$

$y''''(0) = \phi''''(0) \Rightarrow e^x y''' + e^x y^{(3)} + y' + x y'' + y' + e^x y'' + e^x y^{(4)}$

$\Rightarrow y^{(4)} = \cancel{-y - x y' + y' - e^x y''} = -2a_1 \Rightarrow a_4 = \frac{-2a_1}{4!}$

$y^{5} = \frac{e^x y'' + e^x y^{(3)} + e^x y^{(4)} + e^x y^{(5)} + y' + y'' - \;+ y''}{}$

$-a_0 + e^x y^5 + -2a_1 = 0$

$\Rightarrow y^5 = a_0 + 2a_1 \Rightarrow a_5 = \frac{a_0 + 2a_1}{5!}$

Question 7

Consider the equation

$$(x\sin x)y'' + (\cos x)y' + e^x y = 0$$

Check for the two points (i) $x = 0$ and (ii) $x = \pi$ as being ordinary, regular singular or irregular singular points.

for $x = 0 \Rightarrow$

$\lim_{x\to 0} (x) \frac{\cos x}{x \sin x} = \lim_{x\to 0} \frac{\cos x}{\sin x} \Rightarrow \lim_{x\to 0} \frac{\cancel{\sin x}}{\cos x} \frac{1}{0} = \infty$

$\lim_{x\to 0} x^2 \frac{e^x}{x \sin x} = \lim_{x\to 0} \frac{x e^x}{\sin x} \Rightarrow \lim_{x\to 0} \frac{e^x + x e^x}{\cos x} = \frac{1}{1} = 1$

$x = 0$ is irregular sengular point

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Question 8

Find the Laplace transform of the function

$$f(t) = \begin{cases} 0, & 0 \le t < 1 \\ 1, & 1 \le t < 2 \\ 0, & t > 2 \end{cases}$$

$\int_1^2 1\, e^{-st}\, dt \Rightarrow \frac{e^{-st}}{-s}\Big|_1^2 \Rightarrow \frac{-1}{s}\left[e^{-2s} - e^{-s}\right]$

$\frac{-1}{s} e^{-2s} - \frac{e^{-s}}{s}$

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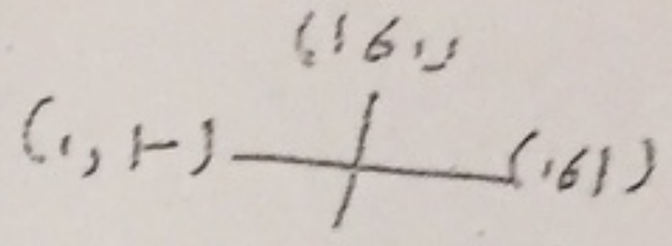

$x = \pi$

1) $\lim\limits_{x \to \pi} (x-\pi) \dfrac{\cos x}{x \sin x} \Rightarrow \lim\limits_{x \to \pi} \dfrac{x \cos x - \pi \cos x}{x \sin x} =$

$\lim\limits_{x \to \pi} \dfrac{\cos x - x \sin x + \pi \sin x}{\sin x + x \cos x} \Rightarrow = \dfrac{-1}{-\pi 1} = \dfrac{1}{\pi}$

$\lim\limits_{x \to \pi} (x-\pi)^2 \dfrac{e^x}{x \sin x} \Rightarrow \lim\limits_{x \to \pi} \dfrac{e^x (x-\pi)^2 + 2(x-\pi) e^x}{x \cos x + \sin x}$

$\Rightarrow \dfrac{0}{\pi} = 0$

π is regular singular point

Question 9

Find the function $y(t)$ whose Laplace transform is the expression

$$\frac{7s^2-8s+12}{(s-2)(s^2+4)}$$

$$\frac{A}{s-2}+\frac{bs+c}{s^2+4} \Rightarrow A(s^2+4)+(bs+c)(s-2)=7s^2-8s+12$$

$$As^2+4A+bs^2-2bs+Cs-2C=7s^2-8s+12$$

$A+b=7 \rightarrow A=7-b \Rightarrow$

$-2b+C=-8$

$4A-2C=12$

$-2b+C=-8$
$4(7-b)-2C=12$

$-2b+C=-8$
$28-4b-2C=12$

$-2b+C=-8$
$-4b-2C=-16$

$+4b-2C=+16$
$-4b-2C=-16$

$\boxed{b=4}$ $\boxed{C=0}$ $4A=12$ $\boxed{A=3}$

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12
16

$A=3 \quad B=4 \quad C=0$

$$\frac{3}{s-2}+\frac{4s}{s^2+4}$$

$\boxed{3e^{2t}+\cancel{2\sin 2t}}$ $4\cos 2t$

Question 10

Use Laplace transform to solve the initial-value problem

$$y''+4y=e^t,\ y(0)=0,\ y'(0)=0$$

$\cancel{s^2Y(s)-sy(0)-y'(0)+4[sY(s)-y(0)]+4Y(s)=\frac{1}{s-1}}$

$\cancel{s^2Y(s)-sy(0)-y'(0)+4sY(s)-4y(0)+4Y(s)=\frac{1}{s-1}}$

$\cancel{(s^2+4s+4)Y(s)=\frac{1}{s-1} \Rightarrow Y(s)=\frac{1}{(s-1)(s^2+4s+4)}}$

$\cancel{Y(s)=\frac{1}{(s-1)(s+2)(s+2)} \Rightarrow \frac{A}{s-1}+\frac{Bs+C}{s^2+4s+4} \Rightarrow \frac{1}{(s-1)(s+2)^2}}$

$$\Rightarrow s^2Y(s)-sy(0)-y'(0)+4Y(s)=\frac{1}{s-1} \rightarrow$$

$$\Rightarrow Y(s)=\frac{1}{(s-1)(s^2+4)}$$

$$\Rightarrow (s^2+4)Y(s)=\frac{1}{s-1}$$

~~b=4 C=0 ⇒ 3~~

$\frac{A}{s-1}+\frac{Bs+C}{s^2+4} \Rightarrow \cancel{A=3}$

~~$3s^2+12+4s+4s^2$~~

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$$\frac{A}{s-1} + \frac{b\cancel{s}+c}{(s^2+4)}$$

$$A(s^2+4)+(b\cancel{s}+c)(s-1) = 1$$

$$As^2 + \boxed{4A} + bs^2 - bs + cs \boxed{-C} = 1$$

$A + b = 0$ $\boxed{C = b}$

$C - b = 0$

$4A - C = 1$

$4A - \cancel{b} = 1$

$A + \cancel{b} = 0$

$5A = 1$ $\boxed{b = -\frac{1}{5}}$

$\boxed{A = \frac{1}{5}}$ $\boxed{C = \frac{1}{5}}$

$$\frac{1/5}{s-1} + \frac{-\frac{1}{5}s + \frac{1}{5}}{s^2+4}$$

$$\frac{1}{5}e^t + \frac{-\frac{1}{5}s}{s^2+4} + \frac{\frac{1}{5}}{s^2+4}$$

$$\frac{1}{5}e^t - \frac{1}{5}\cos 2t + \frac{1}{10}\sin 2t$$