Math 331 Test # 2 First Semester 2013/2014 [redacted]
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Question # 1 (Multiple Choice Question):

(1) Find the Laplace transform of the solution $y(t)$ of the equation:

$$y'-y=e^{-5t},\quad y(0)=2$$

(a) $F(s)=\frac{3}{s(s-5)}$ (b) $F(s)=\frac{2s+11}{s(s+5)}$

(c) $F(s)=\frac{-2s-9}{s(s-5)}$ (d) $F(s)=\frac{2s-9}{s(s+5)}$

$L(y')-L(y)=\frac{1}{s+5}$

$sL(y)-y(0)-L(y)=\frac{1}{s+5}$

$(s-1)L(y)=\frac{1}{(s-1)(s+5)}+\frac{2}{(s-1)}$

$\boxed{L(y)=\frac{2(s+5)+1}{(s+5)(s-1)}}$

$(s+5)(s-1)$

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(2) Find $f(5)$ if $f(t)=2-2u_2(t)+u_\pi(t)-t^2u_6(t)$.

(a) 24 (b) 1

(c) 0 (d) 2

$A=\frac{3\alpha+1}{5}$ $=A(s+3)+B(s-2)$

$B=\frac{-2\alpha+1}{-5}$

(3) Consider the initial-value problem $y''+y'-6y=0,\quad y(0)=\alpha\quad y'(0)=1$. Find the value(s) of α so the $\lim_{t\to\infty} y(t)=0$

(a) 0 (b) $\frac{1}{5}$

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

$s^2L(y)-sy(0)-y'(0)+sL(y)-y(0)-6L$

$s^2L(y)-s\alpha-1+sL(y)-\alpha-6L$

$(s^2+s-6)L(y)=s\alpha+1+\alpha$

$L(y)=\frac{s\alpha+\alpha+1}{(s-2)(s+3)}$

$=\frac{A}{(s-2)}+\frac{B}{(s+3)}$

$\frac{\frac{3\alpha+1}{5}}{s-2}$

$\lim_{t\to\infty}\frac{3\alpha+1}{5}e^{2t}+\lim\frac{-2\alpha+1}{-5}e^{-3t}$

$-\frac{1}{3}$ $\frac{1}{5}-\frac{1}{5}$

(4) Consider the differential equation

$$(x^2-2x+10)y''+y'+(x+1)y=0$$

If a power series solution of the form $\sum_{n=0}^{\infty}a_n(x+3)^n$ converges in some interval about $x_0=-3$, find a lower bound for the radius of convergence of the solution.

(a) 2 (b) $\sqrt{8}$

(c) $\sqrt{13}$ (d) 5

$\rho\geq\min|r-x_0|$

$r^2-2r+10=0$

$\frac{2\pm\sqrt{4-(4)(1)(10)}}{2}=\frac{2\pm\sqrt{-36}}{2}=1\pm\frac{\sqrt{6}}{2}i$

$|3+1\pm\frac{\sqrt{6}}{2}i|$

$|4\pm\frac{\sqrt{6}}{2}i|$

$\sqrt{16+\frac{6}{4}}$

$\sqrt{16+\frac{3}{2}}$

$\boxed{\sqrt{\frac{35}{2}}}$

Find the Laplace transform of the functions in questions (5) – (7)

(5) $f(t) = t\sin\left(\frac{t}{2}\right)$

(a) $\frac{16s}{\left(4s^2+1\right)^2}$ (b) $\frac{2}{4s^2+1}$

(c) $\frac{-8s}{\left(4s^2+1\right)^2}$ (d) $\frac{-16s}{\left(4s^2+1\right)^2}$

(6) $f(t) = u_\pi(t)\cos 2t$

$e^{-\pi s}\, L\left(\cos(2t+\pi)\right)$

$L(\cos 2t)$

$\frac{s}{s^2+4}$

(a) $e^{-\pi s}\left[\frac{2}{(s-1)^2+4}\right]$ (b) $e^{-\pi s}\left(\frac{s}{s^2+4}\right)$

(c) $\frac{s}{(s-1)^2+4}$ (d) $e^{-\pi s}\left(\frac{2}{s^2+4}\right)$

(7) $f(t) = \delta(t-1)\left(2+t^2\right)$

(a) e^{-s}

(c) $3e^{-s}$ (b) $3e^{-2s}$

(d) 2

$f(1)\, e^{-s}$

$2+1$

$3e^{-s}$

(8) The unique solution of the initial-value problem $y' - y = x^2,\ \ y(0) = 1$ has the power series expansion $y = \sum_{n=0}^{\infty} a_n x^n$. Find a_2.

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{3}{2}$ (d) 2

) Classify the point $x = -2$ relative to the differential equation

$x(x+2)y'' + (\sin x)y' + x^2 y = 0$

(a) regular singular (b) irregular singular

(c) ordinary (d) insufficient information

$\lim_{x \to -2} (x+2)\frac{\sin x}{x(x+2)}$

$\lim_{x \to -2} (x+2)^2 \frac{x^2}{x(x+2)} = 0$

Find the inverse Laplace transform of the functions in questions (10) – (13)

(10) $F(s)=\dfrac{1}{(s-2)^2}$

(a) t (b) te^{2t}

(c) e^{2t} (d) t^2e^{2t}

$\frac{1}{(s-2)^2}$ $e^{2t}\,t$

(11) $F(s)=\dfrac{e^{-s}}{s^2+1}$

(a) $\sin t$ (b) $u_1(t)\sin t$

(c) $u_1(t)\sin(t-1)$ (d) $u_1(t)\cos(t-1)$

$u_1(t)$ $\frac{1}{s^2+1}$

(12) $F(s)=\dfrac{4s-3}{s^2-2s}$

(a) $5-3e^{2t}$ (b) $\dfrac{3-5e^{2t}}{2}$

(c) $3+5e^{2t}$ (d) $\dfrac{3+5e^{2t}}{2}$

$\frac{4s-3}{s(s-2)}$

$\frac{4s-3}{s^2-2s+3-3}$

(13) $F(s)=\dfrac{s+7}{(s-1)^2+4}$

(a) $e^t\cos 2t+7e^t\sin 2t$ (b) $e^t\cos 2t+4e^t\sin 2t$

(c) $e^t\cos 2t+3e^t\sin 2t$ (d) $e^t\cos 2t+e^t\sin 2t$

(14) Find the indicial equation for the solution of the equation $(x-1)^2y''+\frac{1}{2}(x-1)y'-y=0$ about the regular singular point $x=1$.

(a) $r^2-\dfrac{r}{2}=0$ (b) $r^2-\dfrac{r}{2}=1$

(c) $r^2+\dfrac{r}{2}=0$ (d) $r^2+\dfrac{r}{2}=1$

$r^2+(\frac{1}{2}-1)r-1=0$

$r^2-\frac{1}{2}r-1=0$

$r^2-\frac{1}{2}r=1$

(15) Find the solution of the equation $ty'-y=0,\ y(0)=1$

(a) $y=e^t$ (b) $y=2t+1$

(c) $y=t+1$ (d) $y=2-e^t$

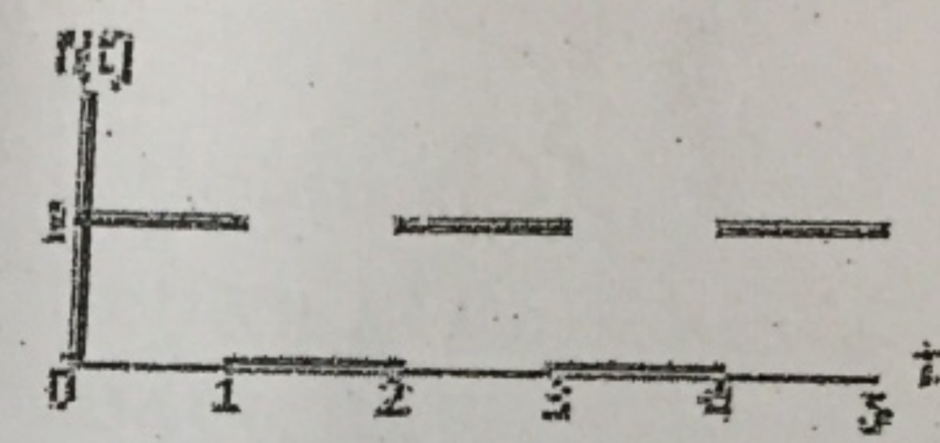
Question # 2

(a) If $f(t)$ is a periodic function with period T, that is, $f(t+T)=f(t)$, for $t\geq 0$, show that

$$\mathcal{L}[f(t)]=\frac{\int_0^T e^{-st}f(t)\,dt}{1-e^{-sT}},\quad s>0$$

(b) Use part (a) above to find Laplace transform of the function graphed below.

$$f(t)=\begin{cases}1, & 0\leq t<1\\ 0, & 1\leq t<2\end{cases},\quad f(t+2)=f(t).\quad t\geq 0$$



$$L(f(t)) = \int_0^\infty e^{-st} f(t)\,dt$$

$$= \int_0^T e^{-st} f(t)\,dt + \int_T^\infty e^{-st} f(t)\,dt$$

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$$= \int_0^T e^{-st} f(t)\,dt + \left(-\frac{e^{-st}}{s} f(t) - \int_T^\infty e^{-st} f'(t)\,dt\right)$$

$$\frac{1}{1-e^{-sT}}$$

$$= \frac{\int_0^T e^{-st} f(t)\,dt}{1-e^{-sT}}$$

$L(f(t)) =$