$\frac{50}{60}$

BIRZEIT UNIVERSITY

Mathematics Department

Ordinary Differential Equations – Math331

Second Exam First Semester 2019 – 2020

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طارق شنك

Section	Instructor	Day	Time
1	Ala Talahmeh	SM	10:00 - 11:15
2	Alaeddin Elayyan	SM	11:25 - 12:40
3	Muna Abu Alhalawa	TR	10:00 - 11:15
4	Abdelrahim Mousa	TR	12:50 - 14:05
5	Ala Talahmeh	TR	11:25 - 12:40
6	Abdelrahim Mousa	MW	10:00 - 11:15
7	Alaeddin Elayyan	SW	12:50 - 14:05

Question One (26 points) Circle the most correct answer:

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1. The particular solution of the DE $y''' + 9y' = 9$ is

   (a) $y_p(t) = 9 + t$

   (b) $y_p(t) = A + B\cos(3t) + C\sin(3t)$

   (c) $y_p(t) = 9$

   (d) $y_p(t) = t$

$r^3 + 9r = 0$
$r(r^2+9) = 0$
$r = 0, \pm 3i$
$y_h = C_1 + C_2 \cos 3t + C_3 \sin 3t$
$y_p = At \rightarrow t$
$y'_p = A$ $\quad 9A = 9$
$y''_p = 0$ $\quad A = 1$
$\boxed{d}$

2. The solution of the DE $xy'' - \frac{2}{x}y = 0,\ x > 0,$ is

   (a) $y(x) = \frac{c_1}{x} + c_2x^2$

   (b) $y(x) = \frac{c_1}{x} + c_2\sqrt{x}$

   (c) $y(x) = c_1 \ln x + c_2x^2$

   (d) $y(x) = c_1 \ln x + c_2\sqrt{x}$

$x^2 y'' - 2y = 0$

$r^2 - r - 2 = 0$
$(r-2)(r+1) = 0$
$r = 2, -1$
$y_h = C_1 x^2 + C_2 x^{-1}$
$\boxed{a}$

3. The method of undetermined coefficients can be used to find particular solution of

   (a) $2y'' - y = \sin x - \sec x$

   (b) $2y'' - y = x + \sin x$

   (c) $2y'' - xy = \sin x$

   (d) $2y'' - y = \frac{1}{x}$

$\sec x = \frac{1}{\cos x}$
$\sin x - \frac{1}{\cos x} = \frac{\sin x \cos x - 1}{\cos x^2}$
$y'' - \frac{1}{2}y = \frac{x}{2} + \frac{\sin x}{2}$
$y'' - \frac{x}{2}y = \frac{\sin x}{2}$
$y'' - \frac{1}{2}y = \frac{1}{2x}$
$\boxed{b}$

$r^2 = \frac{-8 \pm \sqrt{64 - 4(16)}}{2}$ $\quad r^4 + 8r^2 + 16 = 0$

$= -4$ $\quad r = \pm 2i$ $\quad y_p = (AX+B)(C\sin(2x) + D\cos(2x))$

4. The particular solution of the DE $y^{(4)} + 8y'' + 16y = x\sin(2x)$ is

(a) $y_p(x) = (Ax^3 + Bx^2)(C\sin(2x) + D\cos(2x))$
(b) $y_p(x) = (Ax^2 + Bx)(C\sin(2x) + D\cos(2x))$
(c) $y_p(x) = (Ax + B)(C\sin(2x) + D\cos(2x))$ $\quad \times$
(d) $y_p(x) = Ax^2\sin(2x) + Bx^2\cos(2x)$ $\quad \times$

$\boxed{b}$

Euler

5. The general solution of the DE $x^2y'' + 5xy' + 4y = 0,\ x > 0,$ is

(a) $y(x) = \frac{c_1 + c_2 \ln x}{x^2}$ $\quad r^2 + 4r + 4 = 0$
(b) $y(x) = \frac{c_1}{x^2} + \frac{c_2}{x}$ $\quad (r+2)(r+2) = 0$
(c) $y(x) = \frac{c_1}{x^2} + c_2 \ln x$ $\quad r = -2, -2$
(d) $y(x) = c_1 e^{-2x} + c_2 x e^{-2x}$ $\quad y = c_1 x^{-2} + c_2 \ln x \ x^{-2}$

$= \frac{c_1 + c_2 \ln x}{x^2}$ $\quad \boxed{a}$

6. The fundamental set of solutions for the DE $y^{(4)} + 2y''' + 2y'' = 0$ is

(a) $\{1,\ e^{-x}\cos x,\ e^{-x}\sin x\}$ $\quad r^4 + 2r^3 + 2r^2 = 0$ $\quad r = \frac{-2 \pm \sqrt{4-8}}{2}$
(b) $\{1,\ x,\ \cos x,\ \sin x\}$ $\quad r^2(r^2 + 2r + 2) = 0$
(c) $\{1,\ x,\ e^{-x}\cos x,\ e^{-x}\sin x\}$ $\quad = -1 \pm i$
(d) $\{1,\ xe^{-x},\ e^{-x}\cos x,\ e^{-x}\sin x\}$

$y = C_1 + C_2 x + C_3 e^{-x}\cos x + C_4 e^{-x}\sin x$ $\quad \boxed{c}$

$y' = C_2 + C_3 e^{-x}\sin x - C_4 e^{-x}\cos x$

$y'' = -C_3 e^{-x}\cos x - C_4 e^{-x}\sin x$

~~$y''' = -C_3 e^{-x}\sin x + C_4 e^{-x}\cos x$~~ $\quad y''' = ~~C_3 e^{-x}\cos x~~ + C_4 e^{-x}\sin x$

7. Let $y_1 = t$ be the first solution of the DE: $ty'' - y' + ty = 0,\ t > 0.$ The second independent solution is

$y'' - \frac{1}{t}y' + y = 0$

(a) $\ln t$ $\quad W(y_1, y_2)(t) = c e^{\ln t} = ct$
(b) $t\sqrt{t}$
(c) $\sqrt{\ln t}$ $\quad y_2 = y_1 \int \frac{W(y_1, y_2)(t)}{y_1^2}\,dt$
(d) $t\ln t$

$= t\int \frac{ct}{t^2}\,dt = t\int \frac{c}{t}\,dt$

$= t\ln t$ $\quad \boxed{d}$

8. The particular solution of the DE $y'' - 2y' + y = 5e^t$ is

(a) $y_p(t) = Ate^{-t}$ $\quad r^2 - 2r + 1 = 0$
(b) $y_p(t) = Ate^t$ $\quad (r-1)(r-1) = 0$
(c) $y_p(t) = Ae^t$ $\quad r = 1, 1$
(d) $y_p(t) = At^2e^t$ $\quad y_h = c_1e^t + c_2te^t$

$y_p = Ae^t(t^2)$ $\quad$ d $\quad \boxed{d}$

9. The IVP: $(\ln t)y'' + \sqrt{t+2}\,y = \ln t$, $y(2) = 5$, $y'(2) = 1$ is certain to have a unique solution on the interval

$$y'' + \frac{\sqrt{t+2}}{\ln t}\,y = 1$$

$\frac{\sqrt{t+2}}{\ln t}$ cont on $(0, \infty)$

(a) $(1, \infty)$

(b) $(0, \infty)$ ✓

(c) $(-2, 0)$

(d) $(0, 1)$

$\boxed{b}$

10. The solution of the DE $y'' - \alpha y' = 0$ converges as $t \longrightarrow \infty$ if

(a) $\alpha \leq 0$

(b) $\alpha > 0$

(c) $\alpha \in [-1, 1]$

(d) $\alpha \in (-1, 1)$

$r^2 - \alpha r = 0$

$r(r - \alpha) = 0$

$r = 0, \alpha$

$y = C_1 + C_2 e^{\alpha t}$

$\lim_{t \to \infty} y = C_1 + C_2$ when $\alpha \leq 0$

$e^{-t}(1-t)$

$-e^{-t} + (1-t)(-e^{-t})$

$-e^{-t}($

$\boxed{a}$

11. Assume y_1 and y_2 form fundamental solutions of the DE $xy'' - y' + e^x y = 0$, $x > 0$ with $W(y_1, y_2)(1) = 3$, then $W(y_1, y_2)(3) =$

(a) 9

(b) 3

(c) $3e^2$

(d) 1

$y'' - \frac{1}{x}y' + \frac{e^x}{x}y = 0$

$w = c\,e^{\ln x}$

$= cx$

$w(1) = 3$

$3 = c$

$w = 3x$

$\boxed{a}$

$6t + 6$

$3t^2 + 6t$

$-6t + 6$

$-3t^2 + 6t$

$-3 + 6 = 3$

$v' - v = -6t$

$= \frac{dv}{dt} = \frac{dv}{dy} \cdot \frac{dy}{dt}$

$= v\frac{dv}{dy}$

$\frac{dv}{dt} - v = -6t$

12. The particular solution of the DE $y''' + 2y'' + y' = t + e^{-t}$ is

(a) $y_p(t) = At^2 + Bt + Ct^2e^{-t}$

(b) $y_p(t) = At^3 + Bt^2 + Cte^{-t}$

(c) $y_p(t) = At + B + Ce^{-t}$

(d) $y_p(t) = At + B + Ct^2e^{-t}$

$r^3 + 2r^2 + r = 0$

$r(r^2 + 2r + 1) = 0$

$r(r+1)(r+1) = 0$

$r = 0, -1, -1$

$y_{P_1} = (At + B)t$

$= At^2 + Bt$

$y_{P_2} = Ce^{-t}t^2$

$= Ct^2e^{-t}$

$\boxed{a}$

13. Given the IVP: $y'' - y' + 6t = 0$, $y(0) = 1$, $y'(0) = 6$. Then $y(1) =$

(a) 10

(b) 4

(c) 6

(d) 0

$v = y'$ $\quad v' = y''$

$v' - v + 6t = 0$

$v' - v = -6t$

$P(t) = -1$

$q(t) = -6t$

$M(t) = e^{-t}$

$V = e^t\left[\int (e^{-t})(-6t)\,dt + c\right]$

$= e^t\left[-6\int e^{-t}t\,dt + c\right]$

$e^t\left[-6\left[-te^{-t} - e^{-t}\right]\right]$

$6t + 6$

e^{-t} $\quad$ t

$-e^{-t}$ $\quad$ 1

e^{-t} $\quad$ 0

t $\quad$ e^{-t}

1 $\quad$ $-e^{-t}$

0 $\quad$ e^{-t}

$3t^2 + 6t + 1$

$c = 15$

$v = e^t\left[6te^{-t} + 6e^{-t}\right] + c$

$= -t + 1 + c$

$= 1 - t + c$

$= 6 - t$

$6t - \frac{1}{2}t^2 + 1$

$6 - \frac{1}{2} + 1$

Question Two (11 points) Solve the DE

$$y^{(4)} - 2y'' + 3y' - 2y = 0$$

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The aux. Eq: $r^4 - 2r^2 + 3r - 2 = 0$

$(r-1)(r^3 + r^2 - r + 2) = 0$ How

$(r-1)(r+2)(r^2 - r + 1) = 0$ How

$r_1 = 1 \quad r_2 = -2 \quad r_{3,4} = \dfrac{1 \pm \sqrt{1-4}}{2} = \dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2} i$

$r_3 = \frac{1}{2} + \frac{\sqrt{3}}{2} i \quad r_4 = \frac{1}{2} - \frac{\sqrt{3}}{2} i$

$$y_h = C_1 e^{t} + C_2 e^{-2t} + C_3 e^{\frac{1}{2}t} \cos\left(\tfrac{\sqrt{3}}{2} t\right) + C_4 e^{\frac{1}{2}t} \sin\left(\tfrac{\sqrt{3}}{2} t\right).$$

~~$y_h = C_1 e^{t} + C_2 e^{-2t} + e^{\frac{t}{2}} \left[C_3 \cos \frac{\sqrt{3}}{2} t + C_4 \sin \frac{\sqrt{3}}{2} t \right]$~~

~~$y' = C_1 e^{t} - 2C_2 e^{-2t} + e^{\frac{t}{2}} \left[\frac{\sqrt{3}}{2} C_3 \left(-\sin \frac{\sqrt{3}}{2} t\right) + \frac{\sqrt{3}}{2} C_4 \left(\cos \frac{\sqrt{3}}{2} t\right) \right] + \frac{1}{2} e^{\frac{t}{2}} [C_3 \cos \frac{\sqrt{3}}{2} t + C_4 \sin$~~

~~$= C_1 e^{t} - 2C_2 e^{-2t} + \frac{\sqrt{3}}{2} e^{\frac{t}{2}} \left[C_4 \cos\left(\frac{\sqrt{3}}{2} t\right) - C_3 \sin\left(\frac{\sqrt{3}}{2} t\right) \right]$~~

~~$y'' = C_1 e^{t} + 4C_2 e^{-2t} +$~~

$r^4 - 2r^2 + 3r - 2 = 0$

$(r-1)(r^3 + r^2 \quad)$

Question Three (9 points) Solve the IVP

$$y^2y'' + (y')^3 = 0\,, \qquad y > 0\,, \qquad y(0) = 1\,, \qquad y'(0) = \frac{1}{2}$$

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let $v = y'$ $\quad v' = y''$

$v = \frac{dy}{dt}$

$y^2 v' + v^3 = 0$

$\frac{dv}{dt} = \frac{dv}{dy} \cdot \frac{dy}{dt} = v \frac{dv}{dy}$

$y^2 v \frac{dv}{dt} + v^3 = 0$

$y^2 \frac{dv}{dy} + v^2 = 0$

$y^2 dv = -v^2 dy \Rightarrow \int \frac{dv}{v^2} = -\int \frac{dy}{y^2}$

$\boxed{\frac{-1}{v} = \frac{+1}{y} + C}$

$v = \frac{1}{2}$, $t = 0$, $y = 1$

$-2 = 1 + C$

$\boxed{C = -3}$

$v = \frac{1}{2}$, $t = 0$, $y = 1$ $\quad -3 = C \Rightarrow \frac{-1}{v} = \frac{1}{y} - 3$

$\frac{1}{v} = 3 - \frac{1}{y} \Rightarrow \frac{1}{v} = \frac{3y-1}{y} \Rightarrow v = \frac{y}{3y-1}$

$\int \frac{dy}{dt} = \int \frac{y}{3y-1} \Rightarrow \int \frac{3y-1}{y} dy = \int dt$

Continue.

$$\int 3-\frac{1}{y}\,dy=\int dt$$

$$3y-\ln y=t+C_2$$

$y=1$
$t=0$

$$3-\ln(1)=0+C_2$$

$$\boxed{3=C_2}$$

$$\boxed{3y-\ln y=t+3}$$

Question Four (11 points) Solve the DE

$$x^2y'' - 2xy' + 2y = \frac{1}{x} \quad , \qquad x > 0$$

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~~$y'' - \frac{2}{x}y' + \frac{2}{x^2}y = \frac{1}{x^3}$~~

~~$P(t) = \frac{-2}{x}$~~

~~$W(y_1, y_2)(x) =$~~

~~The aux Eq: $r^2 - 3r + 2 = 0$~~ Euler

$(r-2)(r-1) = 0$

$r = 2, 1$

$y_h = C_1 e^{2t} + C_2 e^{t} = C_1 t^2 + C_2 t$

$y_1 = t^2 \qquad y_2 = t$

~~$W(y_1, y_2)(t) = \begin{vmatrix} e^{2t} & e^{t} \\ 2e^{2t} & e^{t} \end{vmatrix} = e^{2t+1} - 2e^{2t+1} = -e^{2t+1}$~~

$W(y_1, y_2)(t) = \begin{vmatrix} t^2 & t \\ 2t & 1 \end{vmatrix} = t^2 - 2t^2 = -t$

$y_g = C_1 y_1 + C_2 y_2 + V_1 y_1 + V_2 y_2$

$V_1 = -\int \frac{y_2\, g(t)}{W(y_1, y_2)(t)}\, dt = -\int \frac{t \frac{1}{t^3}}{-t}\, dt = -\int$

اكمال خلف الورقة

$$x^2 y'' - 2xy' + 2y = \frac{1}{x} \quad , x > 0$$

homog: $x^2 y'' - 2xy' + 2y = 0$

The aux: $r^2 - 3r + 2 = 0$

$(r-2)(r-1) = 0$

$r = 2, 1$

$y_h = C_1 x^2 + C_2 x$

$y_1 = x^2 \quad , y_2 = x$

$w(y_1, y_2)(x) = \begin{vmatrix} x^2 & x \\ 2x & 1 \end{vmatrix} = x^2 - 2x^2 = -x^2$

nonhomog: $y'' - \frac{2}{x} y' + \frac{2}{x^2} = \frac{1}{x^3}$

$g(x) = \frac{1}{x^3}$

$v_1 = -\int \frac{y_2 \, g(x)}{w(y_1, y_2)(x)} dx = -\int \frac{x \left(\frac{1}{x^3}\right)}{-x^2} dx = \int \frac{1}{x^4} dx$

$= \frac{x^{-3}}{-3} = \frac{-1}{3x^3}$

$v_2 = \int \frac{y_1 \, g(x)}{w(y_1, y_2)(x)} dx = \int \frac{x^2 \left(\frac{1}{x^3}\right)}{-x^2} = \int \frac{-1}{x^3} dx$

$= \frac{x^{-2}}{2} = \frac{1}{2x^2}$

$y_g = C_1 y_1 + C_2 y_2 + v_1 y_1 + v_2 y_2$

$= C_1 x^2 + C_2 x + \left(\frac{-1}{3x}\right) + \frac{1}{2x}.$

$y' - y = 1$

Question Five (8 points) Use Picard's iteration to solve the IVP $r - 1 = 0 \Rightarrow r = 1$ y_h

$$y' = 1 + y, \qquad y(0) = 0$$

$y' - y = 1$

$y' - y = 0$

homogs aukc $r - 1 = 0$

$r = 1$

$y_h = C_1 e^t$

$y' - y = 1$

nonhomog: $y_p = A$

$y_g = c_1 e^t + A$

$y(0) = 0$

$\boxed{0 = c_1 + A}$ ①

$y_g' = C_1 e^t$

$y' = 1 + y$

$\cancel{C_1 e^t} = 1 + \cancel{C_1 e^t} + A$

$\boxed{A = -1}$

$\boxed{C_1 = 1}$ from ①

$y_g = e^t - 1$

Good Luck