Chapter two: first order D.E's

General forms:- $\frac{dy}{dt} = f(t, y)$ 1st order D.E

$\frac{dy}{dt} = g(t)(h(y))$ seperable 1st order Eq

$\neq 0$

In this Chapter you will learn How to Integrate Differential Equations with the Methods:-

- [x] Standard form Method
- [x] Seperable Equations
- [x] Bernouli's Equation
- [ ]
- [ ]

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II- Standard form Method

- General form:-

$$\boxed{\begin{array}{l} \text{If } \dfrac{dy}{dt} + P(t)\,y = q(t) \\ \text{Then } y = \dfrac{1}{\mu(t)}\left[\displaystyle\int q(t)\,\mu(t)\,dt + c\right] \end{array}}$$

Ex: Outline Q20:-

$$t\,\frac{dy}{dt} + (t+1)\,y = t \qquad y(\ln 2) = 1 \qquad t > 0$$

1- Change the equation to become Similar to the General form

$$\frac{dy}{dt} + \left(\frac{t+1}{t}\right) y = 1$$

2- Now, determine $P(t)$, $q(t)$ and $\mu(t)$

$$P(t) = \frac{t+1}{t}$$

$$q(t) = 1$$

$$\mu(t) = e^{\int\left(\frac{t+1}{t}\right)dt} = e^{\int\left(1+\frac{1}{t}\right)dt}$$

$$= e^{t+\ln t}$$

$$\mu(t) = e^{t+\ln t}$$

$$y = \frac{1}{e^{(t+\ln t)}}\left[\int e^{t+\ln t}\,dt + c\right]$$

$$y = \frac{1}{e^{(t+\ln t)}}\left[\int e^{t}\,t\,dt + C\right]$$

3- Integrate $\int e^t \, t \, dt$

$$\begin{array}{ll} t & e^t \\ 1 & e^t \\ 0 & e^t \end{array}$$

$+$ $-$

$$\int e^t \, t \, dt = te^t - e^t$$

4- put it in the Equation

$$y = \frac{1}{e^t t}\left[te^t - e^t + c\right]$$

$$y = 1 - \frac{1}{t} + \frac{c}{e^t t}$$

4- to find c put :- $y(\ln 2) = 1$

$t = \ln 2$, $y = 1$

$$1 = 1 - \frac{1}{\ln 2} + \frac{c}{2\ln 2}$$

$$0 = \frac{-1}{\ln 2} + \frac{c}{2\ln 2}$$

$\boxed{2 = C}$

2 2.2: Separable Equations

- In this Section, Equations can be separated put x in a side and the y in the other Side

- Outline Q 11

$$x\,dx + y e^{-x}\,dy = 0$$

$y(0)=1$

put $y=1$
$x=0$

$$x\,dx = -y e^{-x}\,dy$$

$$x e^{x}\,dx = -y\,dy$$

$$\int x e^{x}\,dx = \int -y\,dy$$

$$e^{x}(x-1) + C = -\frac{y^2}{2}$$

$$2\left[e^{x}\cdot(x-1) + C\right] = -y^2$$

$$\pm\sqrt{-2\left[e^{x}(x-1) + C\right]} = y$$

put $y=1$ and $x=0$

$$1 = \pm\sqrt{-2[-1 + C]}$$

$$1 = -2[-1 + C]$$

$$1 = 2 - 2C$$

$$-1 = -2C \Longrightarrow \boxed{C = \frac{1}{2}}$$

$$y = \pm\sqrt{-2e^{x}(x-1) + -1}$$

2.4 Second part : Bernouli's Equation

$$\frac{dv}{dt} + P(t)\,V = q(t)\,V^n \quad \text{--- } ①$$

- You will get the equation from The Question on the form:-

$$\frac{dy}{dt} + P(t)\,y = q(t)\,y^n$$

$\boxed{1}$:- Outline Q28 2.4

$$t^2\frac{dy}{dt} + 2ty - y^3 = 0$$

$$\frac{dy}{dt} + \frac{2}{t}y - \frac{y^3}{t^2} = 0$$

① --- $\frac{dy}{dt} + \frac{2}{t}y = \frac{y^3}{t^2}$ $\qquad n=3$

let :- $y = V^{\frac{1}{1-3}}$

$$\boxed{y = V^{-\frac{1}{2}}}$$

② --- $\frac{dy}{dt} = -\frac{1}{2}V^{-\frac{3}{2}}\frac{dV}{dt}$

Equal ① with ②

$$-\frac{2}{t}y + \frac{y^3}{t^2} = -\frac{1}{2}V^{-\frac{3}{2}}\frac{dV}{dt}$$

$$\frac{dV}{dt} = \frac{4}{t}yV^{\frac{3}{2}} + -2\frac{y^3V^{\frac{3}{2}}}{t^2} \qquad \text{put } y = V^{-\frac{1}{2}}$$

$$\frac{dV}{dt} = \frac{4}{t}V + \frac{-2}{t^2}$$

$$\frac{dV}{dt} - \frac{4V}{t} = \frac{-2}{t^2}$$

* $p(t) = \frac{-4}{t}$

* $q(t) = \frac{-2}{t^2}$

* $\mu(t) = e^{\int -\frac{4}{t} dt}$

$= t^{-4}$

So:- $V = \frac{1}{t^{-4}} \left[\left(\int t^{-4} \left(\frac{-2}{t^2}\right) dt + c \right) \right]$

$= t^4 \left[\frac{-2 t^{-5}}{-5} + C \right]$

$V = t^4 \left[\frac{2}{5} t^{-5} + C \right.$

$V = \frac{2}{5} t^{-1} + t^4 C$

$\frac{1}{y^2} = \frac{2}{5} t^{-1} + t^4 C$

$y = \pm \sqrt{\frac{1}{\frac{2}{5} t^{-1} + t^4 C}}$

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