5.1 Review of Power Series

Differentiation and integration of Power Series

Diff: $f(x) = \sum_{n=0}^{\infty} a_n (x-x_0)^n$

$$f'(x) = \sum_{n=1}^{\infty} n a_n (x-x_0)^{n-1}$$

$\vdots$

بتضل n؟

Int: $\int f(x)\,dx = \int \sum_{n=0}^{\infty} a_n (x-x_0)^n \, dx = \frac{a_n (x-x_0)^{n+1}}{n+1}$

Remark: • A function that has a Taylor ~~Series~~ expansion about $x = x_0$ with a Radius of Convergence ρ is Called analytic on ~~an~~ the interval of Convergence

• To shift in Sums

$$\sum_{n=0}$$

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5.2: Series Solution Near an Ordinary Point- Part 1

$P(x)y'' + Q(x)y' + R(x)y = 0$ — * Homog. D.E

If $P(x)$, $Q(x)$, $R(x)$ are Polynomials

- Ordinary Point x_0 :- $P(x_0) \neq 0$
- Singular Point x_0 :- $P(x_0) = 0$

→ Ordin and Sing Pts Can be Complex or Real Points

* To Solve * assume Sol is:-

step $\boxed{1}$ $y = \sum_{n=0}^{\infty} a_n (x-x_0)^n$

Interval of Conv $|x - x_0| < \rho$

↳ Radius of Convergence

step $\boxed{2}$ * Usually we Take $x_0 = 0$ for Simplicity

step $\boxed{3}$ → find Recurrence Relation :- $a_{n+1} = \frac{a_n}{2}$ for Ex is a R.R

step $\boxed{4}$ After finding Solutions We find $W(y_1, y_2)$ and Make Sure It's not = to zero (Independant)

5.3: Series Solutions Near an Ordinary Point Part 2

if you have the equation:-

$$P(x)\, y'' + Q(x)\, y' + R(x)\, y = 0$$

and $P(x)$, $Q(x)$, $R(x)$ are not polynomials

Then to solve it:

Step1: assume $\boxed{a_n = \frac{\phi^{(n)}(x_0)}{n!}}$ and $\phi(x) = y$

Step2: find $a_0, a_1, \ldots$ by substituting the interval Conditions

or by substituting in the main Equation

Step3: $y = a_0 + a_1 x + a_2 x^2 + \cdots$ ← Case $x_0 = 0$

$$= \phi(x_0) + \phi'(x_0)(x - x_0) + \frac{\phi''(x_0)(x - x_0)^2}{2!} + \cdots$$

How to find Radius?

- find Singular Points :-

$\rho = \min \{\rho_1, \rho_2\}$ found by distance between Roots and Center

find where P and Q are non analytic

5.4. Regular and irregular Points

Consider: $P(x)y'' + Q(x)y' + R(x)y = 0$

x_0 is a singular Point $(P(x_0) = 0)$

Make sure That $\lim_{x \to x_0} (x - x_0)\, p(x)$ and $\lim_{x \to x_0} (x - x_0)^2\, q(x)$ are finite

$\frac{Q(x)}{P(x)}$ $\frac{R(x)}{P(x)}$

Both limits should be finite

→ Now if P, Q, R are not poly's

→ Check where p, q are analytic

→ and if p, q are finite Then The point is regular

|| (analytic)