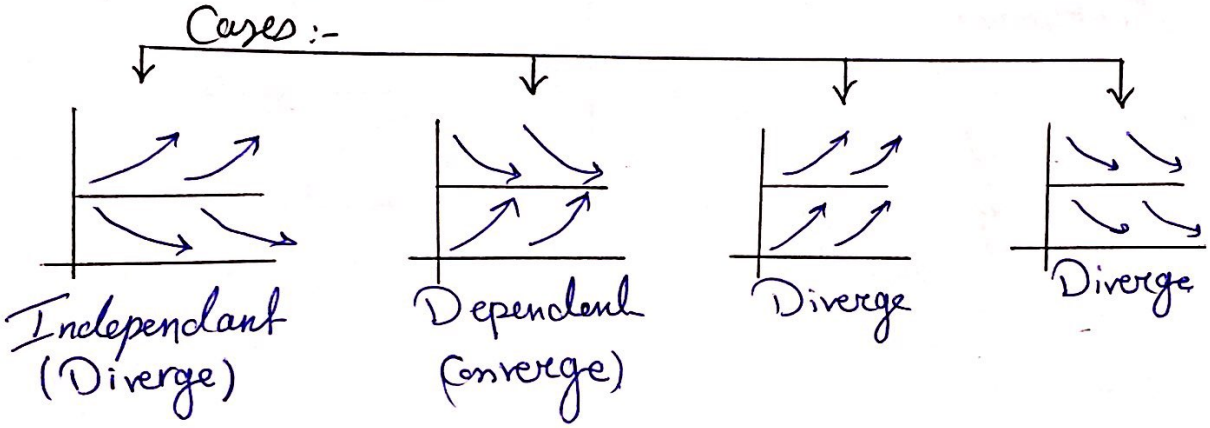
Chapter 1:- Introduction

1.1

- The behavior of the solution for Differential Equation:-
  - ↝ You take the Equilibrium Solution $\Rightarrow \frac{dy}{dt}=0 \Rightarrow y_0$
    Then you choose values for y where once it's higher than y_0 and once less than y_0
  - * If the values of $y > 0 \Longrightarrow$ It's increasing
  - * If the values of $y < 0 \Longrightarrow$ It's decreasing



1.2

- To solve an equation, you integrate after separating

Ex:- 1.2 Problems / Page 16 / Q2 (c)

$$dy/dt = 2y - 10 \qquad y(0) = y_0$$

$$\frac{2}{2}\int \frac{dy}{2y-10} = \int dt$$

$$\ln|2y-10| = 2t + C_1$$

$$2y-10 = \pm e^{2t} e^{C_1}$$

$$2y-10 = e^{2t} C_2$$

Put $t=0$ and $y = y_0$

$$2y_0 - 10 = e^{2(0)} C_2$$

$$2y_0 - 10 = C_2$$

So $\quad 2y-10 = e^{2t}(2y-10)$

$$y = \frac{10 + e^{2t} 2(y-5)}{2}$$

$\Rightarrow y = 5 + e^{2t}(y+5)$

To Graph the behavior:-

Equilibrium:- $\frac{dy}{dt} = 0$

$2y - 10 = 0$

$\boxed{y = 5}$

- If $y < 5 \Rightarrow \frac{dy}{dt} = 2(0) - 10$ (zero for exp)

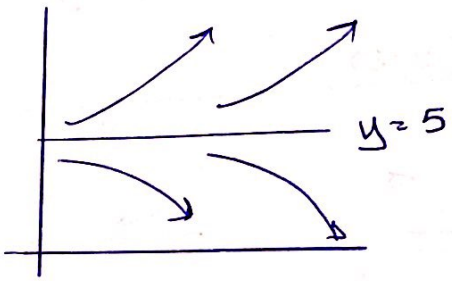
$\frac{dy}{dt} < 0$

so decreasing

- If $y > 5 \Rightarrow \frac{dy}{dt} = 2(6) - 10$

$\frac{dy}{dt} > 0$

so increasing



1.3 Classification of Differential Equation :-

D.E's

Ordinary differential Equation

- Unknown function depends on one independant variable

Ex:- $\frac{dp}{dt} = \frac{1}{2}P - 450$

Partial differential Equation

- Unknow function depends on two or more independant variables

Ex:- $\alpha^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$

$u(t,y)$ function of two variables

System of Differential Equation

- two or more differential Equations

◎ The order of a D.E :-

→ y' : first order

→ y'' : second order

⋮

→ y^n : Nth order

◎ linear And Non linear Equations :-

- first the Constant multipied By y in The function

$F(t, y, y' \cdots y^n) = 0$

should be in terms of (t)

Types of Qustions in Chap 1:-

- IVP:- It gives you the differential EQ and the initial value and you have to find the solution

Ex:- 1.2 outline Q7

$$\frac{dp}{dt} = 0.5P - 450$$

a) $P(0) = 850$ find t at which pop becomes extinct

① find P:- By: separating Equations and Integrating

$$\frac{dp}{dt} = 0.5P - 450$$

$$\int \frac{dp}{0.5P - 450} = \int dt$$

$$\ln|0.5P - 450| = 0.5t + C_1$$

$$0.5P - 450 = \pm e^{0.5t} e^{C_1}$$

$$0.5P = C_2 e^{0.5t} + 450$$

$$P = C_2 e^{0.5t} + 900$$

Put $P = 850$:- $850 = C_3 e^{0.5(0)} + 900$ $\quad |C_3 = C_2/0.5$

$$-50 = C_3 e^{0.5(0)}$$

$$|C_3 = -50|$$

So:- $P = -50 e^{0.5t} + 900$

Now Put P zero and find t

$$\frac{-900}{-50} = e^{0.5t} \implies t = 5.78$$

b) If $P(0) = P_0$ where $0 < P_0 < 900$

But $P = P_0$ when $t = 0$

$$P_0 = C_3 e^{0.5t} + 900$$

$$\boxed{P_0 - 900 = C_3}$$

So $\{P = (P_0 - 900)e^{0.5t} + 900\}$

Now put $P = 0$ and find t

$$\frac{-900}{(-900 + P_0)} = e^{0.5t}$$

$$\ln\left|\left(\frac{-900}{-900+P_0}\right)\right| = 0.5t$$

$$t = 2\ln\left|\frac{-900}{-900+P_0}\right| \quad \text{where } P_0 \neq 900 \text{ and } \frac{-900}{-900+P_0} > 0$$

- Determining Wether The Equation is linear or Non linear and in what order

To know linearity look at two Things :-

1- The power of The Derivative
   - It should Be one

2- The Constant Multiplied By the Derivative
   - It should be in terms of the Independant Unknown

Example:- $2t\frac{dy}{dt}$ - linear / 1st order.

$y'' - e^y y' = 5t$: Non lin / 2nd order