Home work:-

① verify That $y = e^{t^2}\int_0^t e^{-s^2}ds + e^{t^2}$
is a Sol for $y' - 2ty = 1$

$$y' = e^{t^2}\left(e^{-t^2}\right) + \int_0^t e^{-s^2}ds \;\; 2t\, e^{t^2} + 2te^{t^2}$$

$$y' = 1 + \int_0^t e^{-s^2}ds \;\; 2t\, e^{t^2} + 2te^{t^2}$$

$$y' - 2ty = 1 + \boxed{\int_0^t e^{-s^2}ds \;\; 2te^{t^2}} + 2te^{t^2} \; 2t\left(e^{t^2}\int_0^t e^{-s^2}ds + e^{t^2}\right)$$

$$= 1 + \cancel{\int_0^t e^{-s^2}ds \;\; 2t\, e^{t^2}} \cancel{+2te^{t^2}} \cancel{-2te^{t^2}\int_0^t e^{-s^2}ds} - \cancel{2te^{t^2}}$$

$y' - 2ty = 1$

② verify $u(x,y) = \cos x \cosh y$ is a Sol
of $u_{xx} + u_{yy} = 0$

first : u_{xx}

$$\frac{\partial u}{\partial x} = \cosh y\,(-\sin x)$$

$$\boxed{\frac{\partial^2 u}{\partial x^2} = -\cosh y \cos x}$$

$$\frac{\partial u}{\partial y} = \cos x \sinh y$$

$$\frac{\partial^2 u}{\partial y^2} = \cos x \cosh y$$

Adding Them :- $-\cosh y \cos x + \cos x \cosh y = 0$

Home Work 2

① $\frac{dy}{dx} = \frac{xy + 3x - y - 3}{xy - 2x + 4y - 8}$

$\frac{dy}{dx} = \frac{x(y+3) - (y+3)}{x(y-2) + 4(y-2)}$

$\frac{dy}{dx} = \frac{(x-1)(y+3)}{(x+4)(y-2)}$

$dy \frac{(y-2)}{(y+3)} = dx \frac{(x-1)}{(x+4)}$

① $\int \frac{(y-2)}{(y+3)} dy = \int \frac{(x-1)}{(x+4)} dx$ ②

let in ①:-

$+3 + y = u$

$du = dy$

$\int \frac{u-5}{u} du$

$= \int 1 - \frac{5}{u} du$

$= u - 5 \ln u$

$= (y+3) - 5 \ln (y+3)$

let in ②

$4 + x = u$

$du = dx$

$\int \frac{u-5}{u} dx$

$\int 1 - \frac{5}{u} dx$

$= u - 5 \ln u$

$= (4+x) - 5 \ln(4+x)$

So ⇒ $y + 3 - 5 \ln (y+3) = 4 + x - 5 \ln(4+x) + C$

$y - 5 \ln(y+3) = 1 + x - 5 \ln (4+x) + C$

② :- Solve :- $\begin{cases} \dfrac{dy}{dx} = \dfrac{3x^2+4x+2}{2(y-1)} \\ y(0) = -1 \end{cases}$

$$dy\,(2(y-1)) = dx\,(3x^2+4x+2)$$

$$\int 2(y-1)\,dy = \int (3x^2+4x+2)\,dx$$

$$2\left(\frac{y^2}{2} - y\right) = \frac{3x^3}{3} + \frac{4x^2}{2} + 2x + C$$

$$y^2 - \frac{y}{2} = x^3 + 2x^2 + 2x + C$$

$y(0) = -1 \implies$ put $x = 0$, $y = -1$

$$1 + 2 = 0 + 0 + 0 + C$$

$$\boxed{C = 3}$$

Home Work 3

Homogenous Equation:-

$$\left(x^2 \sin\left(\frac{y^2}{x^2}\right) - 2y^2\cos\left(\frac{y^2}{x^2}\right)\right)dx = -2xy\cos\left(\frac{y^2}{x^2}\right)dy$$

$\cos^2$

let $V = \frac{y}{x}$

$y = xV$

$$\frac{dy}{dx} = V + x\frac{dV}{dx} \quad \text{——} (1)$$

$$\left(x^2 \sin\left(\frac{y^2}{x^2}\right) - 2y^2\cos\left(\frac{y^2}{x^2}\right)\right)dx + 2xy\cos\left(\frac{y^2}{x^2}\right)dy =$$

$$\sin\left(\frac{y^2}{x^2}\right) - 2\left(\frac{y^2}{x^2}\right)\cos\left(\frac{y^2}{x^2}\right)dx + \frac{2y}{x}\cos\left(\frac{y^2}{x^2}\right)dy = 0$$

$$\left(\sin(V^2) - 2(V^2)\cos(V^2)\right)dx + 2V\cos(V^2)\,dy = 0$$

$$\sin(V^2) - 2(V^2)\cos(V^2) = -2(V)\cos(V^2)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{\sin V^2 - 2V^2\cos(V^2)}{-2V\cos(V^2)}$$

$$\cancel{V} + x\frac{dV}{dx} = \frac{\tan(V^2)}{-2V} + \cancel{V}$$

$$\int \frac{dV\,(-2V)}{\tan(V^2)} = \int \frac{dx}{x}$$

$$-\ln x + c = \int \frac{1}{v(1+\sin(v^2))} \, dv$$

$u = v^2$

$du = 2v\,dv$

$$\int \frac{-2v}{\tan(v^2)} \, dv = \int \frac{dx}{x}$$

$$= \ln|x| + C$$

$$-\int \frac{du}{\tan(u)}$$

$$-\int \frac{\cos u}{\sin u} \, du = -\ln|\sin u|$$

$$-\ln|\sin u| = \ln|x| + C$$

$$-\ln|\sin(v^2)| = \ln|x| + C$$

$$-\sin(v^2) = x\, e^C$$

$$\sin(v^2) = -x C_2$$

$$v^2 = \sin^{-1}(-x C_2)$$

$$\frac{y^2}{x^2} = \sin^{-1}(-x C_2)$$

Home Work 4:

$$\frac{dy}{dx} - \underbrace{\frac{2y}{(6x+1)}}_{p(t)} = \underbrace{\frac{-3x^2}{(6x+1)y^2}}_{q(t)}$$

$$y = v^{\frac{1}{1-2}}$$
$$= v^{\frac{1}{3}}$$

$$\frac{dy}{d\cancel{t}} = \frac{1}{3}(v)^{-\frac{2}{3}} \frac{dv}{d\cancel{x}}.$$

$$\cancel{\frac{dy}{dx} =}$$

$$\frac{2y}{6x+1} - \frac{3x^2}{(6x+1)y^2} = \frac{1}{3}(v)^{-\frac{2}{3}} \frac{dv}{dx}$$

$$\frac{2v^{\frac{1}{3}}}{6x+1} - \frac{3x^2}{(6x+1)(v^{\frac{2}{3}})} = \frac{1}{3}(v)^{-\frac{2}{3}} \frac{dv}{dx}$$

$$\frac{dv}{dx} = \frac{6v}{6x+1} - \frac{3x^2}{6x+1} =$$

$$\frac{dv}{dx} - \underbrace{\frac{6v}{6x+1}}_{p(x)} = \underbrace{\frac{-3x^2}{6x+1}}_{q(x)}$$

$$\mu(x) = e^{\int \frac{-6}{6x+1}}$$
$$= e^{-\ln|6x+1|}$$

$$\mu(x) = \frac{1}{6x+1}$$

$$v = \frac{1}{1/6x+1}\left[\int \frac{1}{6x+1}\left(\frac{-3x^2}{6x+1}\right)dx + c\right]$$

$$= 6x+1 \left[\int \frac{-3x^2}{(6x+1)^2}\,dx + c\right]$$

$$= 6x+1 \left[\int \frac{-3x^2\,dx}{36x^2+12x+1} + c\right]$$

$$= (6x+1)\left[\int \frac{x^2\,dx}{-12x^2-4x-\frac{1}{3}} + c\right]$$

$$= (6x+1)\left[-3\int \frac{x^2\,dx}{36x^2+12x+1} + c\right]$$

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$$= -3(6x+1)\left[\int \frac{1}{36} + \frac{-\frac{1}{3}x + \frac{1}{36}}{36x^2+12x+1}\right]$$

$$\begin{array}{r} \frac{1}{36} \\ 36x^2+12x+1 \overline{) \; x^2 \qquad\qquad} \\ -\; x^2 + \frac{1}{3}x + \frac{1}{36} \\ \hline -\frac{1}{3}x + \frac{1}{36} \end{array}$$

$$= -3(6x+1)\left(\left[\frac{1}{36}x + \frac{-1}{3}\int \frac{x - \frac{1}{12}}{36x^2+12x+1}\right] + c\right)$$

$$\cancel{= -3(6x+1)\left[\frac{1}{36}x + \frac{-1}{3}\int \frac{1}{36x} + \frac{x}{36x^2+12x+1} - \frac{1}{12(36x^2+12x+1)}\right]}$$

$$= -3(6x+1)\left(\left[\frac{1}{36}x + \frac{-1}{3}\int \frac{x}{36x^2+12x+1} - \frac{1}{12}\ln(36x^2+12x+1) + c\right)\right.$$

$$= -3(6x+1)\left(\left(\frac{1}{36}x - \frac{1}{3} \qquad -\frac{1}{12}\ln(36x^2+12x+1)\right) + c\right)$$

Home Work 5

$(3x^2y - 8x)y' = 4y - 2xy^2$

$(3x^2y - 8x)dy = (4y - 2xy^2)dx$

$\underbrace{(2xy^2 - 4y)}_{M}dx + \underbrace{(3x^2y - 8x)}_{N}dy = 0$

$M = 2xy^2 - 4y \longrightarrow M_y = 2x(2y) - 4$

$N = 3x^2y - 8x \longrightarrow N_x = 6xy - 8$

$\cancel{2(2xy-2)}$

$M_y \neq N_x$

$$\frac{M_y - N_x}{N} = \frac{4xy - 4 - 6xy + 8}{3x^2y - 8x}$$

$$= \frac{-2xy + 4}{3x^2y - 8x} \qquad \frac{-2(xy-2)}{3x(xy-8)}$$

$$= \frac{4 - 2xy}{-8x + 3x^2y}$$

$$= \frac{2(2 - xy)}{-8x + 3x^2y} = \frac{2(2 - xy)}{x(-8 + 3xy)}$$

$$= \frac{2(2 - xy)}{3x(-\frac{8}{3} + xy)}$$

It Didn't Work

Try $\quad \dfrac{M_y - N_x}{M} = \dfrac{-2xy + 4}{2xy^2 - 4y} = \dfrac{-2\cancel{(xy-2)}}{2y\cancel{(xy-2)}} = -1/y$

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So $I(y) = e^{-\int -\frac{1}{y} dy}$

$= y$

$y[(2xy^2-4y)\,dx + (3x^2y-8x)\,dy] = 0$

$(2xy^3-4y^2)\,dx \quad + (3x^2y^2-8xy)\,dy = 0$

$M = 2xy^3-4y^2 \longrightarrow M_y = 6xy^2-8y$

$N = 3x^2y^2-8xy \longrightarrow N_x = 6xy^2-8y$

So It's Exact

$\psi_{(x,y)} = \int \psi_x \, dx = \int 2xy^3-4y^2 \, dx$

$= \frac{2x^2y^3}{2} - 4y^2x + g(y)$

$\psi_{(x,y)} = x^2y^3 - 4y^2x + g(y)$

Now: $\psi_y = (\psi_{(x,y)})'$

$3x^2y^2-8xy = 3x^2y^2-8yx+g'(y)$

$g(y) = 0$

So

$x^2y^3 - 4y^2x = c$ is the Sol