P34: Some Special Second Order Equations:

I. Equations with the dependent variable missing "missing y".

The general form of second order equations is

$$y'' = f(t, y, y')$$

when y is missing, the equation becomes

$$y'' = f(t, y')$$

let $v = y'$, $v' = y'' \Rightarrow v' = f(t, v)$ is 1^{st} order eq.

solving this eq. for v, then we solve the eq.

$\frac{dy}{dt} = v$ to get $y(t)$.

36. $t^2 y'' + 2t y' - 1 = 0, \quad t > 0$

$$y'' + \frac{2}{t} y' = \frac{1}{t^2}$$

let $v = y' \Rightarrow v' + \frac{2}{t} v = \frac{1}{t^2}$ "linear":

$$\mu(t) = e^{\int \frac{2}{t} dt} = e^{2\ln t} = t^2$$

$$v(t) = \frac{1}{t^2}\left[\int \cancel{t^2} \frac{1}{\cancel{t^2}} dt + c\right]$$

$$= \frac{1}{t^2}\left[t + c\right] \Rightarrow v(t) = \frac{1}{t} + \frac{c}{t^2}$$

$$y'(t) = v(t) = \frac{1}{t} + \frac{c}{t^2} \Rightarrow y(t) = \ln t - \frac{c_1}{t} + c_2$$

II. Equations with independent variable missing : missing t.

$$y'' = f(y, y')$$

let $v = y' \Rightarrow \frac{dv}{dt} = y'' \Rightarrow \frac{dv}{dt} = f(y, v)$

this eq. hass too many variables. We think of y as independent variable, then,

$$\frac{dv}{dt} = \frac{dv}{dy}\frac{dy}{dt} = v\frac{dv}{dy}$$

the eq. becomes: $v\frac{dv}{dy} = f(y, v)$

solving this eq. we get v as a function of y

then we solve the seperable equation

$$\frac{dy}{dt} = v(y)$$

from which we get $y(t)$, which is the ~~la~~ solution of the original equation.

42. $yy'' + (y')^2 = 0$

let $v = y'$, $y'' = v' = v\frac{dv}{dy}$,

$yv\frac{dv}{dy} + v^2 = 0$ $\Rightarrow v = 0$ is a solution

$\Leftrightarrow y(t) = c$ is a solution

If $v \neq 0 \Rightarrow y\frac{dv}{dy} + v = 0$

$\Rightarrow \frac{d}{dy}(vy) = 0$

$\Rightarrow v(y)\,y = c_1$

$v(y) = \frac{c_1}{y}$

but $v = \frac{dy}{dt}$

$\Rightarrow \frac{dy}{dt} = \frac{c_1}{y} \Rightarrow \int y\,dy = \int c_1\,dt$

$\frac{y^2}{2} = c_1 t + c_2$