1.1 Mathematical Models; Direc~~tion Fields~~

Differential equations: are equations containing derivatives.

see section 1.3 1^{st} page

→ equations mean relations and

→ derivatives mean rates

To understand problems that involve

- motion of fluids حركة السوائل
- the flow of current in electric circuits دائرة كهربائية
- Population dynamics
- dissipation of heat in solid objects تبديد حرارة في جسم صلب
- detection of seismic waves الكشف عن الموجات الزلزالية

⋮

it's necessary to know differential equations.

* Differential equations that describe a physical process is called mathematical model.

Example Formulate a differential equation describing the motion of an objective falling in the atmosphere near the sea level.

→ Variables : time t independent variable

velocity v dependent variable

→ Physical law governs the motion of objects is Newton's 2^{nd} law : The mass of the object times its acceleration is equal to the net force on the object.

$\Rightarrow \quad F = m\,a$

Note that $a = \dfrac{dv}{dt}$

F : net Force exerted on the object مبذولة

m : mass

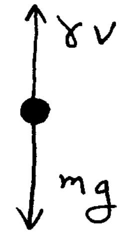
a : acceleration

a : $\dfrac{\text{meter}}{\text{second}^2}$

m : kg

F : newton

$$F = m\frac{dv}{dt} \quad \text{------} (1)$$



* Forces that act on the object :
  - Force of gravity $F = mg$ (m → mass, g → acceleration due to gravity $\left(\approx 9.8\ \frac{m}{sec^2}\right)$)
  - Force of air resistance $F = \gamma v$
    γ is constant called معامل سحب drag coefficient

$F = \gamma\, v$

$\frac{kg\,m}{s^2} = \gamma\,\frac{m}{s}$

* Net Force is $\quad F = mg - \gamma v$

From (1) $\Rightarrow \quad m\dfrac{dv}{dt} = mg - \gamma v \quad \longleftarrow$ mathematical Model of an object falling from atmosphere

* Take $m = 10$ kg , $\gamma = 2$ kg/sec and $g = 9.8\ \frac{m}{sec^2}$ near sea level.

we obtain $\boxed{\dfrac{dv}{dt} = 9.8 - 0.2\,v}$ $\rightarrow$ *

قوانين نيوتن الثلاثة

① يظل الجسم في حالته الحركية (إما السكون أو الحركة في خط مستقيم بسرعة ثابتة) مالم تؤثر عليه قوة تغير من هذه الحالة . $\sum F = 0$

② إذا أثرت قوة أو مجموعة قوى في جسم فإنه تكسبه تسارع a يتناسب مع محصلة القوى المؤثرة ، ومعامل التناسب هو كتلة القصور الذاتي m للجسم

$$\sum F = ma$$

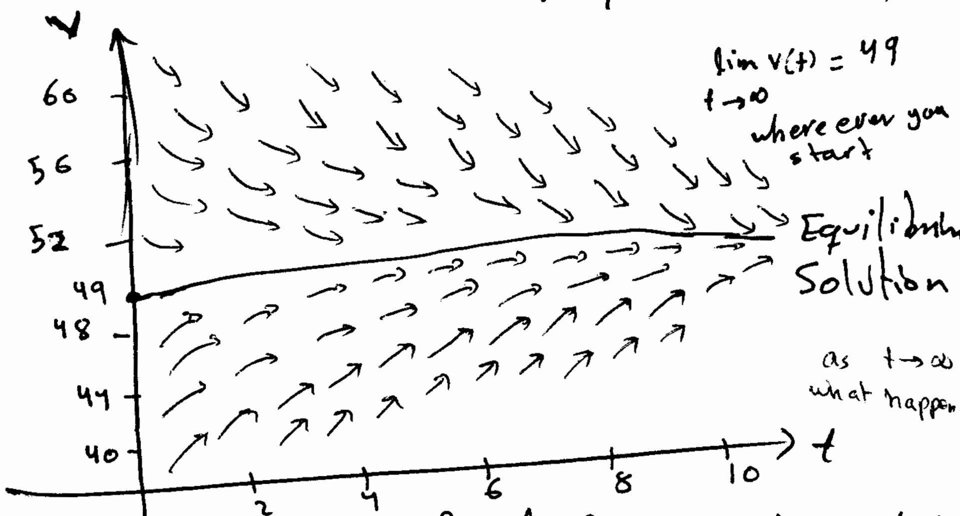
③ لكل قوة رد فعل مساوي له في المقدار ومعاكس له في الاتجاه

To solve $*$, we need to find the solution $v = v(t)$ that satisfies the equation.

→ This is not hard, we will learn how to find such solution.

→ To sketch the direction field (behavior of the solution), we use the differential equation $*$ to build a table: (values of v do not depend on t)

v	$\frac{dv}{dt}$
0	9.8
5	8.8
10	7.8
15	6.8
20	5.8
30	3.8
50	-0.2
60	-2.2



direction field for $*$ using Maple (MatLab) ...

$*$ The horizontal solution curves are called equilibrium solutions.

If we make $v' = \frac{dv}{dt} = 0$ the $*$ becomes $9.8 = 0.2\,v$

$v = 49$

$*$ Equilibrium Solutions: In general: For a differential equation of the form $y' = ay - b$ $^{a\neq 0}$, to find the equilibrium solution, we set $y' = 0$ and we solve for y.

$y(t) = \frac{b}{a}$ $\quad a \neq 0$

Example Find the Equilibrium solutions of

① $y' = 2 - y$ ② $y' = y(y+2)$

① $y' = 0 \Rightarrow y(t) = 2$

② $y' = 0 \Rightarrow y(y+2) = 0 \Rightarrow$ either $y(t) = 0$ or $y(t) = -2$

//

Example : Mice and Owls الفئران والبوم

Assume that a mouse population increases at a rate proportional to the current population in the absence of predators (owls).
مفترسين

→ let us denote time by t in months and the mouse population by $p(t)$

The population growth is given by

$$\frac{dp}{dt} = r p \quad \text{where} \quad \text{----} (1)$$

r is the growth rate or the rate constant.

→ assume that $r = 0.5$ mice/month. Then (1) has units of mice/month

→ when owls are present, they eat the mice. Suppose owls eat 15 per day. Write a differential Equation describing the mouse population in the presence of owls.

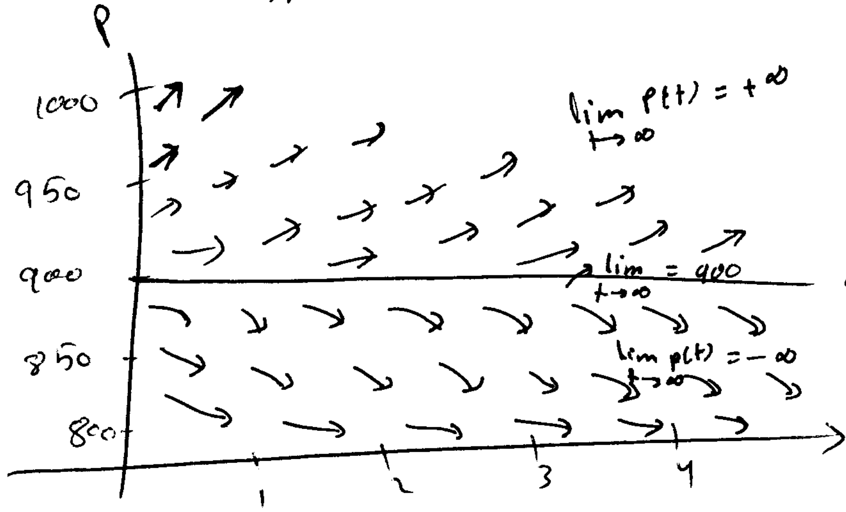
$$\Rightarrow \frac{dp}{dt} = 0.5p - 450$$

الزيادة في الشهر — النقصان في الشهر

$p' = 0 \Rightarrow$

$0.5p = 450$

$p = 900$



Equilibrium Solution

as $t \to \infty$ what happens

Direction field "behavior of solution"

Now its important to Find anaylitic solution instead of drawing ...

H.W page 8 : 1, 7, 11, 22, 23, 25