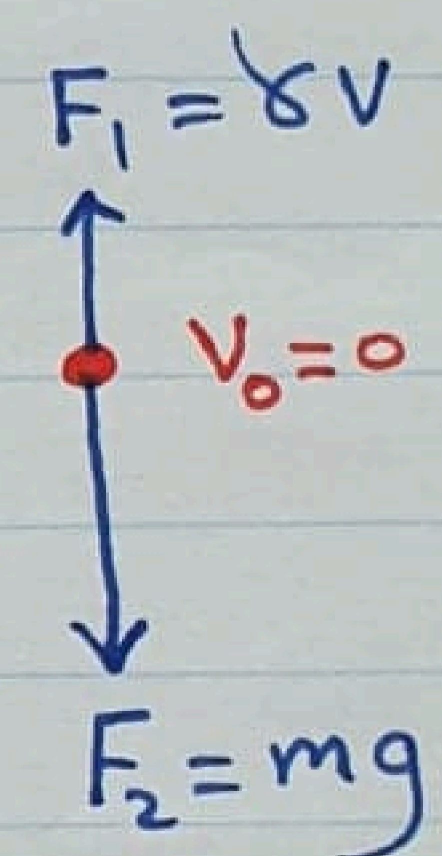
Modeling with DE's

- Sometimes DE's are called Mathematical Models (MM) as they can use to model (describe) a physical phenomena (waves, heats, velocity of objects, ...) or chemical phenomena (radioactivity, reactions, ...) or biological phenomena (population growth / Decay, ...) or psychological phenomena (Process of making decisions, ...)

Exp (Free Fall)

An object falls from a rest

① Write MM describing its velocity change over time



m : mass of object
$V(t)$: velocity of object at time t
γ : drag coefficient
$g \approx 9.8\ m/sec^2$ acceleration due to gravity

Net force is $\Delta F = F_2 - F_1$

$$ma = mg - \gamma v$$

a: acceleration
$a = v'(t) = \frac{dv}{dt}$

$$m\frac{dv}{dt} = mg - \gamma v$$

$$\boxed{\frac{dv}{dt} = g - \frac{\gamma}{m} v} \rightarrow MM$$

② Take $m = 10\ Kg$, $\gamma = 2\ Kg/sec$, $g \approx 9.8\ m/sec^2$
Find Eq. Sol.

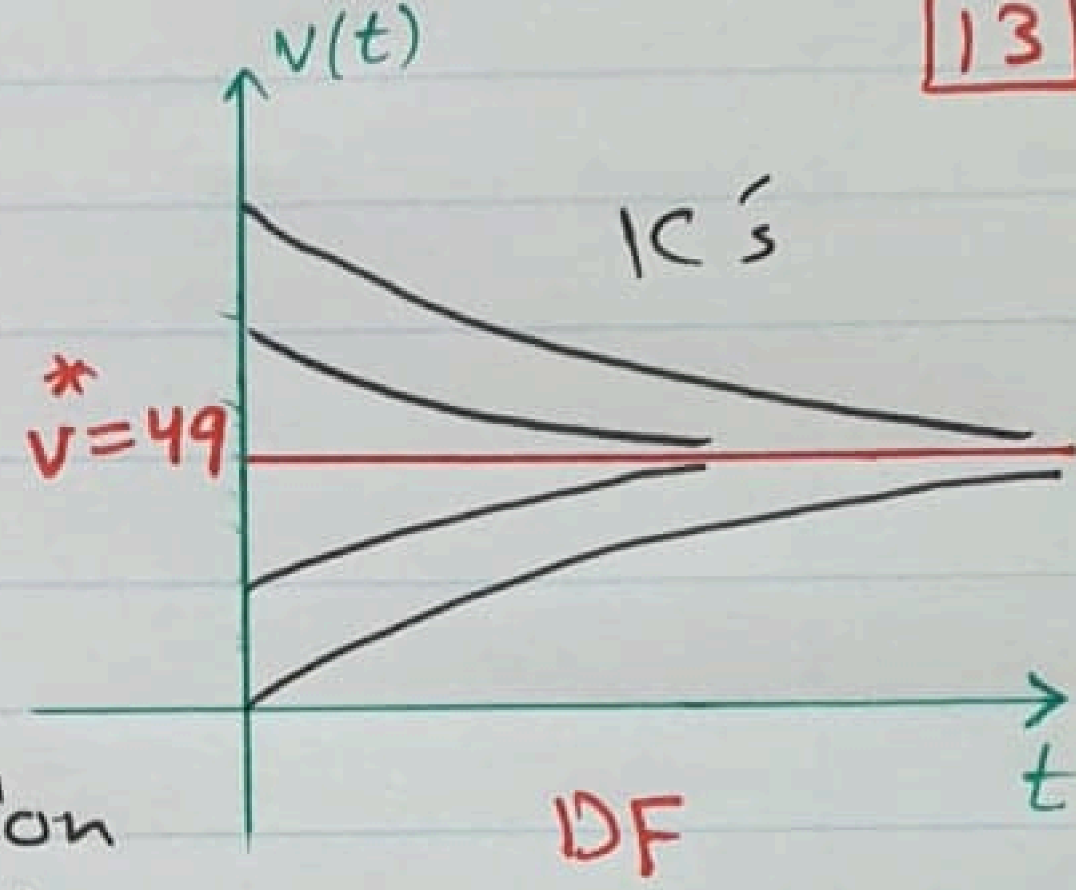
$$\frac{dv}{dt} = 9.8 - 0.2\,v \quad \Rightarrow \frac{dv}{dt} = 0 \quad \Rightarrow v^* = \frac{9.8}{0.2} = 49$$

[3] Draw the DF

$$\acute{v} = 9.8 - 0.2v$$

if $v_0 = 100$ then $\acute{v} < 0 \Rightarrow v(t) \downarrow$

if $v_0 = 0$ then $\acute{v} > 0 \Rightarrow v(t) \uparrow$



[4] study the behavior of solution
"Find the limiting velocity"

$$\lim_{t \to \infty} v(t) = 49$$

$a = -0.2 < 0$

$v(t) \to 49$

<u>Exp</u> (Mice and Owls)

Assume a mice population $p(t)$ increases proportionally to its current size, where t in months.

① write MM to describe the change in $p(t)$ over time

$$\frac{dP}{dt} \propto P$$

$$\frac{dP}{dt} = rP$$ where the constant r is called the growth rate $(r>0)$.

② Assume the owls (predators) come and they eat 15 mice/day. Write MM to describe the change in $p(t)$ over time (consider $r = 0.5$)

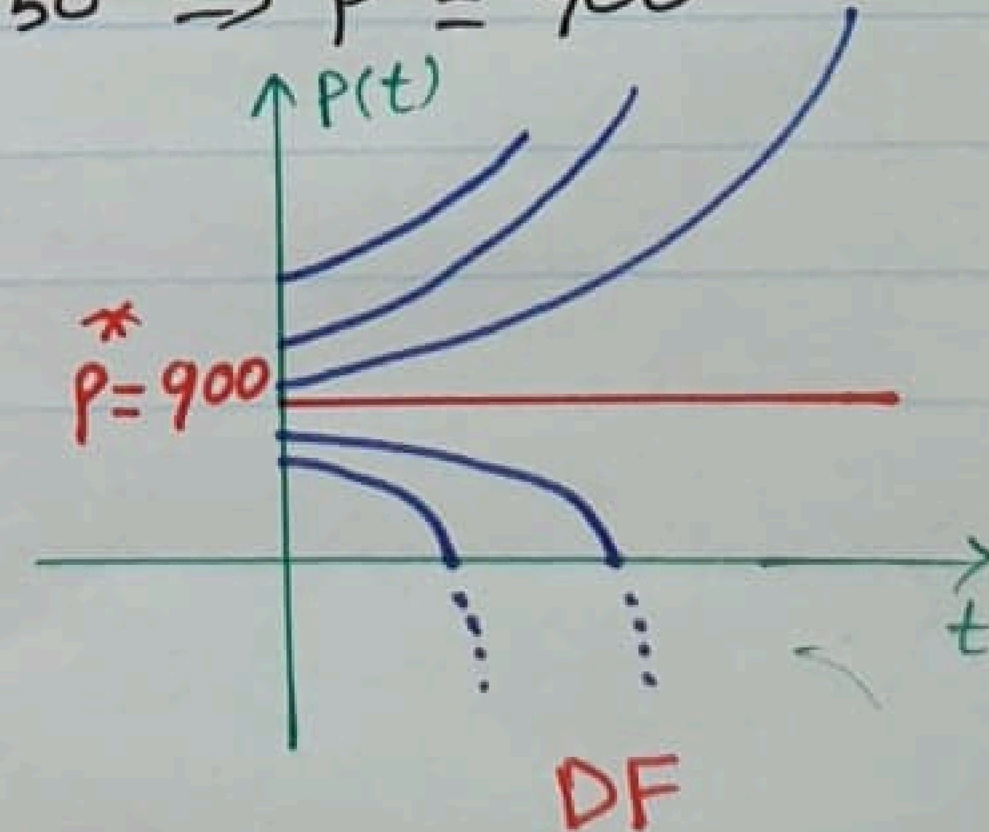
$$\frac{dP}{dt} = 0.5P - 450$$

15×30 since t in months

③ Draw the DF and study the behavior of solution

Eq. Sol. $\Rightarrow \frac{dP}{dt} = 0 \Rightarrow 0 = 0.5P - 450 \Rightarrow P^* = 900$

$$\lim_{t \to \infty} P(t) = \begin{cases} \infty & \text{if } P_0 > 900 \\ 900 & \text{if } P_0 = 900 \\ \boxed{0} & \text{if } P_0 < 900 \end{cases}$$



Exp Assume $\phi(t)$ is a solution for the IVP:

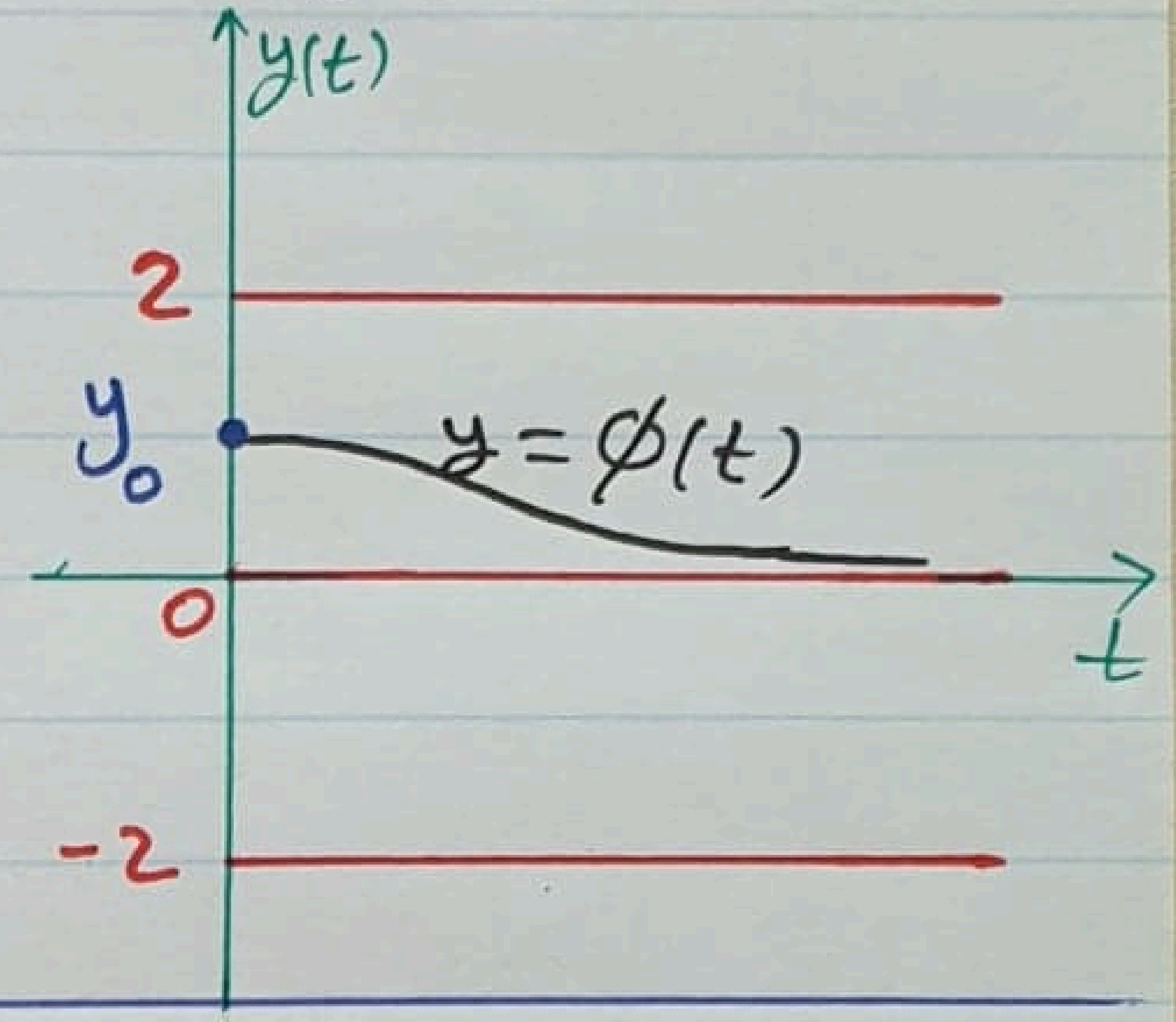
$$y' = y(y^2-4), \quad y(0)=1$$

Find $\lim_{t\to\infty} \phi(t)$

Eq. Sol. $\Rightarrow y' = 0 \quad \Rightarrow y^* = 0, 2, -2$

If $y_0 = 1 \Rightarrow y' = -3 < 0 \Rightarrow y(t) \downarrow$

$$\lim_{t\to\infty} \phi(t) = 0$$



Three important questions for a given DE

1) Is there a solution?

2) If there is a sol., is it unique?

3) If there is a sol., how to find it?

- Now we will start solving DE's instead of drawing the DF
- The solution we get by solving a DE is called analatical solution