Remark Assume y^* is an Eq. Sol. for the DE

$$y' = f(y) \quad \text{---} (1)$$

① If $f'(y^*) > 0$, then y^* is unstable Eq. Sol.

② If $f'(y^*) < 0$, then y^* is asymptotically stable Eq. Sol.

③ If $f'(y^*) = 0$, then y^* is semistable Eq. Sol.

Exp Find Eq. Sol's and classify them for the DE

① $y' = y(4-y^2) \Rightarrow y' = 0 \Rightarrow y(2-y)(2+y) = 0$

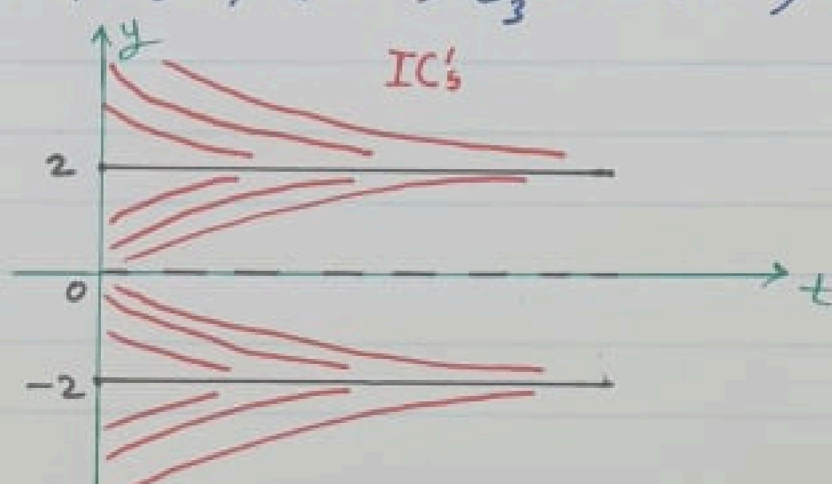
$\Rightarrow y_1^* = 0, \; y_2^* = 2, \; y_3^* = -2$

Compare with (1) $\Rightarrow$ **Eq. Sol**

$y' = f(y) = y(4-y^2) = 4y - y^3$

$f'(y) = 4 - 3y^2$

- $f'(y_1^*) = f'(0) = 4 > 0 \Rightarrow y_1^* = 0$ is unstable Eq. Sol.
- $f'(y_2^*) = f'(2) = 4 - 3(4) < 0 \Rightarrow y_2^* = 2$ is asymptotically Eq. Sol.
- $f'(y_3^*) = f'(-2) = 4 - 3(4) < 0 \Rightarrow y_3^* = -2$ is asymptotically Eq. Sol.



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② $y' = y^2 \Rightarrow y' = 0 \Rightarrow y^* = 0$ Eq. Sol.

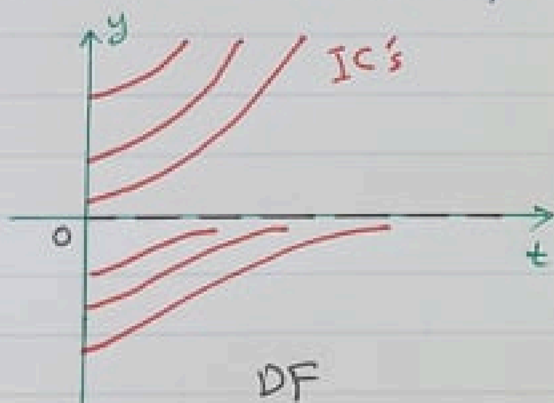
Compare with (1) $\Rightarrow$

$y' = f(y) = y^2 \Rightarrow f'(y) = 2y$

$f'(y^*) = f'(0) = 0 \Rightarrow y^* = 0$ is semistable Eq. Sol.

if $y_0 = 1 \Rightarrow y' > 0 \Rightarrow y(t) \uparrow$

if $y_0 = -1 \Rightarrow y' > 0 \Rightarrow y(t) \uparrow$



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Notes ① If y^* is asymptotically stable Eq. Sol., then the IC's gets closer to y^* from both sides.

② If y^* is unstable Eq. Sol., then the IC's gets far away from y^* from both sides.

③ If y^* is semistable Eq. Sol., then the IC's gets closer to y^* from one side and becomes far away from y^* from the other side.